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INTRODUCTION TO THE MEDITATIONS OF DESCARTES.

Translated from the French by WM. R. WALKER.

[To the readers of Descartes' Meditations, a translation of which appeared in vol. iv. of the Journal, the following introduction and epitome will prove interesting.—EDITOR.]

To the Deans and Doctors of the Sacred Faculty of Theology of Paris.

GENTLEMEN:—The reason which leads me to present to you this work is so just (and, when you know its design, I am assured that you will have also as just a reason for taking it under your protection), that I think I cannot do better, in order in some sort to recommend it to you, than to tell you in a few words what my purpose is. I have always considered that the two questions of God and of the Soul were preeminently those which ought rather to be demonstrated by philosophical than by theological reasons; for although it suffices us who are of the faith to believe by faith that there is a God, and that the human soul does not die with the body, it certainly does not seem possible ever to persuade infidels of the truth of any religion, or scarcely even of any moral virtue, if we do not first prove to them these two things by natural reason; and inasmuch as there are often offered in this life greater rewards for vices than for virtues, few persons would prefer the just to the useful were they not restrained either by the fear of God or by the expectation of another life: and although it be absolutely true that it is necessary to believe that there is a God because it is so taught in the Holy

Scriptures, and on the other hand that it is necessary to believe the Holy Scriptures because they come from God (the reason being that as faith is a gift of God, he himself who gives grace to enable us to believe other things can also enable us to believe that he exists), yet we could not propose that to infidels, who would imagine that we had thereby committed the fault which logicians call a circle.

And, moreover, I have observed that you, gentlemen, with all the theologians, have asserted not only that the existence of God can be proved by natural reason, but also that the knowledge of him is much more clear than that which we have of many created things, and indeed that it is so simple that those who have it not are guilty; as appears by those words of Wisdom, chap. xiii., where it is said that "their ignorance is unpardonable: for if their minds have penetrated so far into the knowledge of the things of the world, how did they not sooner find out the Lord thereof?"—and in Romans, chap. i., it is said that they are "inexcusable," and again in the same place, by these words, "that which is known of God is manifest in them," it would seem we are informed that all which may be known of God can be shown by reasons which need not be drawn elsewhere than from ourselves and from the simple consideration of the nature of our mind. For this reason I have believed that it will not be against the duty of a philosopher to show here how and by what means we can, without going beyond ourselves, know God more easily and more certainly than we know the things of the world.

And as regards the soul, although many have believed that it is not easy to know its nature, and some have even ventured to say that human reason convinces us that it dies with the body, and that it is only faith which teaches us the contrary; yet, since the Lateran Council, held under Leo X., Sess. 8, condemns these, and expressly enjoins on Christian philosophers to answer their arguments, and to employ all the powers of their mind to make the truth known, I have ventured on the undertaking. Moreover, knowing that the principal reason which makes many impious persons unwilling to believe that there is a God, and that the human soul is distinct from the body, is, as they say, that no one has hith-

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erto been able to demonstrate these two things; and though I am not of their opinion, but on the contrary hold that the majority of the reasons brought by so many great persons concerning these questions are, well understood, so many demonstrations, and that it is almost impossible to discover new ones; hence I believe one can do nothing more useful in philosophy than at once to investigate with care the best things, and to place them in so clear and exact an order that it will henceforward be unquestionable to every one that they are veritable demonstrations. And, in a word, since many persons have desired it of me who know that I have cultivated a certain method for resolving all kinds of difficulties in the sciences, a method which indeed is not new (there being nothing more ancient than truth), but which they know I have used to good advantage on other occasions, I have thought it my duty also to make proof of it in a matter so important.

Now I have labored to the best of my ability to include in this treatise all that I have been able to discover by means of that method. Not that I have here collected all the various reasons which could be alleged by way of proof on so great a subject: for I have never believed that to be necessary, unless in cases where there is nothing certain; but have only treated of the first and principal ones in such a manner that I venture to offer them as very evident and certain demonstrations. And I will say, moreover, that they are such, that I do not think there is any way by which the human mind can ever discover better; for the importance of the subject, and the glory of God to which all this has reference, constrain me to speak here rather more freely of myself than has been my custom. Nevertheless, whatever certainty and evidence I find in my reasons, I cannot persuade myself that every one is capable of understanding them. But, just as in geometry there are many things which have been bequeathed to us by Archimedes, by Appollonius, by Pappus, and by many others, which are received by every one as very certain and evident because they contain nothing which, considered separately, is not very easy to be known, and because everywhere the things which follow have an exact connection with and dependence upon those which precede; yet, because they are somewhat long and require an undivided mind, they are

comprehended and understood by only a very few: in the same way, while I consider those which I here make use of, to equal or even surpass in certainty and evidence the demonstrations of geometry, I nevertheless apprehend that by many they cannot be sufficiently understood, partly because they also are somewhat long and dependent the one upon the other, and chiefly because they require a mind entirely free from all prejudices and which can easily detach itself from commerce with the senses. And, truth to say, there are not in the world so many fit for the speculations of metaphysics as for those of geometry. And, moreover, there is yet this difference that, in geometry, each being aware of the opinion that nothing is there advanced which has not with certainty been demonstrated,—those who are not entirely versed oftener transgress in approving of false demonstrations, in order to make believe they understand them, than in refuting the true. It is not thus in philosophy, where, every one believing that all is there problematical, few give themselves up to the investigation of truth; and very many, wishing to acquire the reputation of bold spirits, strive after nothing else than arrogantly to combat the most apparent truths.

This, gentlemen, is why, whatever force my reasons may have, I do not hope, since they belong to philosophy, that they will produce a great effect on men's minds, if you do not take them under your protection. But the esteem in which your body is held by the whole world being so great, and the name of the Sorbonne being of such authority, that not only in that which concerns the faith has there ever, after the sacred Councils, been so much deference paid to the judgment of any other body; but also, in what concerns human philosophy, every one believing that it is not possible to find elsewhere more solidity and knowledge, or more prudence and integrity in giving judgment, I do not doubt, if you deign to take so much care of this writing as to be willing first to correct it (for, having knowledge not only of my infirmity, but also of my ignorance, I would not dare to assert that there are no errors), then to add to it the things which are wanting, to finish those which are not perfect, and to take the trouble of giving a fuller explanation to those parts which may have need of it, or at least to inform me of these in order that I may

make the explanation; and, in short, after the reasons by which I prove that there is a God and that the human soul is different from the body, shall have been brought to that point of clearness and evidence to which I am assured they may be conducted, that they will be held as very exact demonstrations, if you deign to stamp them with your approbation and give public testimony to their truth and certainty,—I do not doubt, I say, that after that, all the errors and false opinions which have ever existed concerning these two questions will soon be effaced from the minds of men. For the truth will so work that all the learned and the men of understanding will subscribe to your judgment and authority; the atheists, who are generally more arrogant than learned and judicious, will be robbed of their spirit of contradiction, or perhaps will themselves defend the reasons which they will find received by all persons of understanding as demonstrations, lest they should appear not to have intellect; and, finally, all others will readily yield to so many proofs, and there will not be any person who will venture to doubt of the existence of God, and of the real and veritable distinction between the human soul and the body.

It is, however, for you to judge of the fruit which will be produced by this confidence, if it were once well established, —you who see the disorders which doubt produces; but I should not here act with a good grace were I further to recommend the cause of God and of religion, of which you have always been the firmest supports.

PREFACE.

I have already touched on the two questions of God and of the human soul in the Discourse, written in French, which appeared in the year 1637, concerning the method for the proper conduct of reason and for the search of truth in the sciences; not with the design of then treating them fundamentally, but only by the way, in order to discover, by the judgment passed upon them, in what manner I should afterwards treat them: for to me they have always seemed to be of such importance that I judged it fitting to speak of them oftener than once; and the course which I take in order to explain them is so

little beaten and so far removed from the ordinary route, that I have not considered it would be profitable to present it in French, and in a discourse which could be read by all, lest weak minds should suppose that it is permitted to them to attempt this road.

But, having in the Discourse on Method begged all who should find in my writings anything worthy of censure, to do me the favor of apprising me of it, there has been no objection of any note made, save to two things on which I had written concerning these two questions, to which I shall here respond in a few words before undertaking their more precise explanation.

The first objection is, that while the human mind reflecting on itself does not know itself as anything but a thing that thinks, it does not thence follow that its nature or its essence is only to think,—in such a manner that the word *only* excludes all other things which one could perhaps speak of as belonging to the nature of the soul.

To which objection I answer that it was not my intention in that place to exclude them so far as concerns the truth of the thing (of which I was not then treating), but only as concerns the order of my thought; my meaning being that I was not acquainted with anything which I knew to belong to my essence, except that I was a thing that thinks, or a thing which has in it the faculty of thinking. But I shall afterwards show how, that if I know nothing else to belong to my essence, it thence follows that nothing else really does belong to it.

The second objection is, that if I have in me the idea of a thing more perfect than myself, it does not thence follow that this idea is more perfect than I, and still less that what is represented by this idea exists.

But I answer that in this word *idea* there is here a double meaning: for it may either be taken in a material sense for an operation of my understanding, and in this sense it could not be said that it is more perfect than myself; or it may be taken objectively for the thing which is represented by this operation, which, although it is not supposed to exist out of my understanding, can nevertheless be more perfect than I by reason of its essence. But in the course of this treatise I shall show more fully how, from this alone, that if I have in

me the idea of a thing more perfect than I, it thence follows that this thing actually exists.

Moreover, I have also seen two other sufficiently copious writings on this matter, but these do not so much attack my reasons as my conclusions, and that by arguments drawn from the common-places of atheists. But, because arguments of that kind can make no impression on the minds of those who understand my reasons aright, and because the judgments of many are so weak and so little reasonable that they are oftener persuaded by the first opinions they may have had of a thing, however false and removed from reason these may be, than by a solid and real but subsequently brought refutation of their opinions, I do not wish in this place to answer them, lest I should be obliged first to repeat them.

I shall only say in general that all that the atheists say in combatting the existence of God depends always either on what they imagine to be in God of human affections, or on their attributing to our minds so much power and wisdom that we have verily the presumption of wishing to determine and comprehend what God can and ought to do; so that all that they say need give us no trouble, provided only we remember that we must consider our minds as things finite and limited, and God as a being infinite and incomprehensible.

Now, after having sufficiently recognized the sentiments of men, I again undertake to treat of God and of the human soul, and at once to lay the foundations of the first philosophy, but without expecting any praise from the vulgar, or hoping that my book will be read by many. On the contrary, I shall never counsel any to read it, except those who wish to meditate with me seriously, and who can detach their minds from the commerce of the senses, and deliver themselves entirely from prejudices of every kind; those persons I know to be but very few in number. But for those who, without caring much for the order and the connection of my reasons, will amuse themselves in criticising the separate parts, as many do,—those, I say, will not find great profit in the reading of this treatise; and though they may perhaps find in many places occasion to cavil, they will have great difficulty in making any weighty objection, or any that would be worthy of answer.

And though I do not promise to others to satisfy them at first, and do not esteem myself so highly as to believe that I can foresee what will be each one's difficulty, I shall first reveal in these Meditations the very thoughts that led me to a certain and evident knowledge of the truth, in order to see whether, by the same reasons which persuaded me, I shall be able to persuade others; and thereafter I shall answer the objections which have been made to me by persons of understanding and erudition to whom I sent my Meditations for the purpose of being examined before they were sent to the press; for they have made so many and so different objections, that I dare promise myself that it will be difficult for another to bring forward anything important which has not been already touched upon.

This is the reason for which I beg those who desire to read these Meditations not to form any judgment until they shall first have taken the pains to read all the objections and the answers which I have made to them.

EPITOME OF THE SIX MEDITATIONS.

In the first, I bring forward the reasons by which we may doubt generally of all things, and particularly of material things, at least as long as we have no other foundations in the sciences than those we have hitherto had. But although the utility of a doubt so general may not at first be apparent, it is, however, very great in this respect, that it frees us from all manner of prejudices, and prepares for us a very easy way of accustoming our minds to detach themselves from the senses; and in this, that it makes it impossible for us ever to doubt of the things which we shall thereafter discover to be true.

In the second, the mind which, using its proper liberty, supposes that nothing exists, concerning whose existence it has the least doubt, discovers that it is nevertheless absolutely impossible that it does not itself exist. This also is very useful, inasmuch as by this means it easily makes distinction between the things which pertain to it—that is to say, to the intellectual nature—and the things which pertain to the body.

But, because it may happen that some will expect of me in this place reasons to prove the immortality of the soul, I deem it right here to warn them that having aimed at stating nothing in all this treatise of which I shall not have very exact demonstration, I am thus obliged to follow an order similar to that made use of by geometricians, which is to first advance all the things on which the proposition sought for depends, before drawing any conclusion.

Now, the first and principal thing requisite in order rightly to know the immortality of the soul is to form a clear and precise conception of it entirely distinct from all the conceptions we may have of the body; which has in this place been done. It is besides requisite to know that all things which we clearly and distinctly conceive are true after the fashion in which we conceive them; this could not be proved before the fourth Meditation. Moreover, it is necessary to have a distinct conception of the corporeal nature, which conception is given partly in the second, and partly in the fifth and sixth Meditations. And, in fine, we must conclude from all this that the things which we clearly and distinctly conceive to be diverse substances, such as the mind and the body, are in fact substances really distinct from one another, and such is the conclusion drawn in the sixth Meditation; this is again confirmed, in the same Meditation, by the fact that we do not conceive any body but as divisible, whereas the mind or the soul of man cannot be conceived but as indivisible; for, in fact, we could not conceive the half of any soul, as we can of the smallest of all bodies; and hence we discover that their natures are not only diverse, but in a fashion contrary. But I have not further treated of this matter in this work, partly because that will show with sufficient clearness that from the corruption of the body the death of the soul does not ensue, thus giving to men the hope of a second life after death, and partly because the premises from which we conclude the immortality of the soul depend on the explanation of all physics: in the first place, to know that generally all substances—that is to say, all things which cannot exist without being created by God—are in their nature incorruptible, and that they can never cease to be, unless God himself, refusing his concurrence, reduce them to nothing; and then, to observe

that the body taken in general is a substance, and cannot therefore perish; but that the human body, inasmuch as it differs from other bodies, is but composed of a certain configuration of members and of other similar accidents, whilst the human soul is not thus composed of any accidents, but is a pure substance. For although all its accidents are changed —for example, although it conceives of certain things, wills others, and feels others, etc.—the soul nevertheless does not become another; whereas the human body becomes another thing, from this alone that the figure of some of its parts becomes changed: whence it follows that the human body can very easily perish, but that the mind or soul of man (I do not distinguish between the two) is immortal in its nature.

In the third Meditation, I have, it seems to me, explained at sufficient length the principal argument of which I make use in order to prove the existence of God. But nevertheless, because I did not wish to make use in this place of any comparisons drawn from corporeal things, in order to withdraw as far as I could the mind of the reader from the usage and commerce of the senses, there may perhaps remain many obscurities here (which, as I hope, will be entirely cleared up in the answers I have given to the objections that have since been proposed to me), for instance, among others, this one: how the idea of a Being supremely perfect, which we find within us, contains so much objective reality—that is to say, participates by representation in so many degrees of being and of perfection, that it must come from a supremely perfect cause. This I have made clear in these answers by the comparison of a very ingenious and artificial machine, the idea of which is found in the mind of some artisan; for, as the objective product of this idea must have some cause, namely, either the knowledge of this artisan or of that of some one from whom he has received this idea, so also is it impossible that the idea of God, which is in us, has not God himself for its cause.

In the fourth, it is proved that all things which we conceive very clearly and very distinctly are every one true; and at the same time is explained in what consists the nature of error or falsity, which must necessarily be known, as much

to confirm the truths preceding as for the better understanding of those which follow. But there is yet this to remark, that I do not at all there treat of sin—that is to say, of error which is committed in the pursuit of good and evil—but only of that which occurs in the judgment and discernment of the true and the false; and that I am not to be there understood as speaking of the things which belong to the faith or to the conduct of life, but only of those which regard speculative truths, and which can be known only by the aid of the natural light alone.

In the fifth Meditation, besides corporeal nature, taken in general, being explained, the existence of God is also there demonstrated by a new reason, in which however some difficulties may be also encountered, but the solution will appear in the answers to the objections which have been made; and, in addition, I show in what manner it is true that the certainty even of geometrical demonstrations depends on the knowledge of God.

Finally, in the sixth, I distinguish between the action of the understanding and that of the imagination; the marks of this distinction are there described; I show that the soul of man is really distinct from the body, and yet that the two are so closely conjoined and united that they seem to compose but one thing. All the errors which proceed from the senses are there exposed, with the means of avoiding them; and, in a word, I bring forward all the reasons by which we can conclude of the existence of material things; not that I judge them of much use in proving what they prove, namely, that there is a world, that men have bodies, and other similar things, which have never been held as doubtful by any man of good sense; but because in considering them closely one comes to know that they are not so well established or so evident as those which lead us to the knowledge of God and of our soul; inasmuch as these are the most certain and evident which can come to the knowledge of the human mind, and this is all I have had the design of proving in these six Meditations, and on this account I have here omitted many other questions of which I have incidentally spoken in this treatise.

KANT'S ETHICS.

By James Edmunds.

III.—*Ethico-active Reason.*

§ 25. Man (homo phenomenon) knows himself to be possessed of a faculty (wherewith he knows) which does not seem to belong to the world of nature (mundus sensibilis). This he cogitates to be a FACULTY, knowing himself obliged to its use and exercise. But since no supersensible faculty could possibly belong to a solely sensuous existence (thing), he thinks this faculty as (quasi) belonging to a supersensible man (person, homo noumenon), an intelligent. This boldest dictum he formulates in the apodictic supersensible-axiom of DES CARTES, "cogito, ergo sum."

§ 26. That of which this unquestionable deliverance of consciousness declares the being, is neither body nor soul (for a demonstrated world of souls can be no other than a sensible world). It is the philosophic EGO, of which we predicate intelligible existence. We declare him to be an intelligent, the possessor of the faculty reason. That is all.

§ 27. Speaking after the analogy of the world (for in truth an intelligent, having no external relations, can form no part of a congeries, much less of a world, and therefore cannot act), the EGO is said to belong to a world (mundus intelligibilis, a convenient philosophic fiction) and to be in that world an agent. His rule is the law of reason, which immediately determines the will, no foreign determinator (mobile) being possible. But no sooner does he enter into the phenomenal world than he finds himself in his phenomenal (external) character subjected to the law of that world, mechanical necessity, which thrusts itself upon his will and (if not resisted) violently determines it.

§ 28. Now if he (homo phenomenon) were absolutely subjected to natural necessity, it were absurd to speak of his will as free, since it must be always externally determined. If again he, being in the natural world, were absolutely subjected to the law of reason (an intelligent in the world but not of it, homo noumenon among phenomena,—an absurd supposition), it were equally absurd (at least superfluous) to

speak of his will as free; since, though independent of natural determination, it is by hypothesis subjected to self and admits but one determination, ethic necessity. But in actual fact and event he knows himself independent on natural necessity, since he may enforce self-determination in the very face of natural necessity; and (as it were) equally independent on his own reason, since he may (if he so will) degrade himself to the purely animal level and resolutely deny to reason her rightful command: in other words, he recognizes in himself that curious compound, free humanity, which is of the very essence of two worlds yet compelled in neither, which may elevate itself at will toward the higher or degrade itself in the lower, which builds its own heaven or digs its own hell.

§ 29. But because the rational agent is free to permit the determination of his will by either the mechanic necessity which appertains to his phenomenal character or the moral necessity which is the law of his intelligible character, it must not be concluded that he is independent of both worlds (dependent on neither). Any such definition of freedom must be excluded from philosophic terminology, for the reason that the conception possesses no value whatever (not to insist that it is not true, insomuch as it represents the agent neither as intelligent nor as phenomenon nor as a compound of the two, but as some incogitable nondescript).

§ 30. The philosophic conception of freedom is simply independence on foreign determinators. And since ethico-rational necessity is (so far as can be concluded upon rational ground) no other than proper (absolutely internal) determination, the conception freedom of will is equivalent to subjection to the moral law, which last is but self law, reason cogitated as efficient in form of law and acting according to the representation of law. He who represents freedom, whether in the schools or in the state, as lawlessness, is laughed at.

§ 31. Speculative reason is reason (in her character of ratio phenomenon, the only character in which she views herself to herself objected, and so KNOWS herself) beholding herself, and endeavoring by the aid of her indubitable fact consciousness to retrace the processes of the workings of her thought, thereby discovering for the behoof of her finite phenomenal self her veritable and absolute ethical self.

§ 32. Practical reason is reason (in her character of FACULTY, ratio noumenon, the only character in which she can cogitate herself as possessed of energy) phenomenizing herself, obtruding her rule into the sensible world and therein enforcing its behest, most marvellously transmuting the beautiful but uncurrent metal of intellect into the glory of golden fact. Art beyond alchemy! Wonderful of incarnations, incarnate reason! He who dare enter into her holiest of holies, finds there her fires never dim.

§ 33. Ethics is the science of right and wrong. The notion of an act in general (of which it is not considered whether it is right or wrong) is divided according to a rule of reason into that which is right and that which is wrong. Ethics is therefore the science of the law of reason, i. e. (since we have no knowledge of any law not of reason) of law in general; and ethico-active reason is reason acting according to law, i. e. her own law.

§ 34. Here the careless reader will stumble at once upon the apparent antinomy of the law (freedom and nature), which does not yet concern us. For the groundwork of all science, we go back of the sensible into the intelligible world. Now if of any idea we do not make complete abstraction, we are liable in deductions to the error of undistributed middle, a logical snare set at every turn for philosophy. We have previously (§ 14) called attention to the fact that an apriori deduction of morality is habitually demanded by those who have never strictly attempted to divest morality of a completely aposteriori character; and the very difficulty upon which we have paused is due to the remarkable circumstance that we deceive ourselves, persuading ourselves that we have made abstraction of reason from the rational agent, the while nevertheless unconsciously cogitating it as a THING which exists SOMEWHERE and SOMEWHEN. (The ground of abstraction, which underlies the simplest generalization as well as the most ultimate apodict, is deep hid in the very nature of the faculty. We cannot ask why reason is reason, but only what reason is.)

§ 35. The error to which we point is far more extensive and of more serious consequence than may hastily appear. Indeed it is the occasion of the fact to which Mr. KANT refers

(in the transcendental dialectic, commenting upon ARISTOTLE) in remarking that "it is nothing unusual, in common conversation as well as in written works, by comparing the thoughts which an author has delivered upon a subject, to understand him better than he understood himself; insomuch as he may not have sufficiently determined his conception, and thus have sometimes spoken, nay even thought, in opposition to his own opinions." But the mistake occurs often where the conception has been sufficiently determined, and nevertheless the mind is suffered by a sort of carelessness to disregard the determination. And this we take to be the origin and explanation of most of the contradictions into which Mr. KANT has himself apparently fallen. Since the complete definition of the terminology is, especially in such argument as we have approached, of vital importance, it may be well to suggest a few specifications:

§ 36. We understand by noumenon a substance plus the totality of its possible (the totality of the possibility of its) accidents or manifestations: the totality of its real accidents, existent as given through sensibility, we call phenomenon. We think to the phenomenon the same substance as to the noumenon; but (owing to the subjective nature of our representations) we cannot know what (if anything) in the phenomenon, except the substance, belongs also to the noumenon. Noumenon as "an object of a non-sensuous intuition," is an impossibility; for we cannot conceive of a noumenon which might not (in some or other of its modes) become phenomenon, if sensibility were so modified as to receive (intuite) it. "Non-sensuous intuition" is a contradiction in terms; and whatever intuition we think, other than that which we possess, is not non-sensuous, but differently sensuous. The noumenon is therefore no possible object; and the conception of the homo noumenon is that of a mere schema.

§ 37. It will not be understood that we are asserting substance of the EGO, in which we cognize nothing. The permanent in time (substantia phenomenon, matter) is nothing more than a schema of substance, through which we reach as it were to grasp very substance. That complex of mere relations which we call matter, is an attempt to cogitate substance as an entity in the real world and therefore as neces-

sarily possessed of internal determinations in relation to itself (of extensive relations).

§ 38. So that complex of internal determinations which we call EGO, is an attempt to cogitate man (abstract from the real world) as possessed of relations in the intelligible world. But since the determination of the homo noumenon is absolutely internal, and not relatively (in time as opposed to space, which internal is truly external), the relations are no more than a bold figure; and the complex is an empty ethical schema, no more real than a sensible schema.

§ 39. The real is that which exists in sensation. Now nothing can be known experimentally to exist, except it be given in sensation: hence reality is taken for existence, and unreality logically becomes non-existence; while in truth unreality is also unrealized ideality. Reality in time is merely the schema of existence; and to declare even a thing unreal is not to deny its existence.

§ 40. Abstract from time, substance is —— substance! Mr. KANT'S attempt to define it as that which must be cogitated as subject and never as predicate,* will not bear scrutiny; for we have already begun by predicating it of its accidents. This confusion may have arisen from not clearly distinguishing subject logical (as opposed to predicate) from subject actual (as opposed to object); but, although substance cannot be cogitated as object, to define it as subject in nowise increases the clearness of the conception, but on the contrary is liable to confuse it by leading to its use as undistributed middle.

§ 41. A faculty acts, and must be able to act, else it were not faculty: in other words, to act is involved in the very notion of faculty, and is a mere deduction. (Not that a faculty must act continuously.)

The action of a faculty is as to its mode more properly termed function. Thus: will is a function of reason; but will is itself cogitated as a faculty when it is viewed as reason acting rather than as an act of reason.

These punctual distinctions are essential to any specula-

* See pages 79, 92, 113, 174, 181, 183, 193, 226, 241, 243, 260, Meiklejohn's Kritik, Bohn's edition.

tion. For example, synthesis of apprehension in nowise differs from the transcendental synthesis of imagination, nor this from the synthetic unity of apperception upon which all synthesis is founded. But in synthesis of apprehension we cogitate only so much of the activity of consciousness as is indispensably necessary to give unity to the empirical intuition; and we not so much view the result with reference to its unity of the manifold as that we seize upon its manifold in unity. Different names for different phases of the same act draw the attention more or less to the action itself, as the argument may require.

§ 42. Particular faculties of the mind are merely phases of the activity of the EGO, distinguished for purposes of convenient reference. Now when Mr. KANT explains (in the deduction of the categories) that imagination belongs partly to the sensibility, he seems to us merely fall into a confusion of terms. Mr. TUPPER was not far from right when he said that "Imagination is not sense, neither is fancy reflection." Imagination and sensibility are both faculties of the EGO, and both necessary the one to the other (for cognitive purposes); but the one no more belongs to the other than the other to the one. And Mr. KANT'S argument to prove that the unity of formal intuition apriori belongs to space and time and not to the conception of the understanding, is nothing but the logical result of confusion and possesses no value.

§ 43. Reason is the sole faculty which an intelligent is cogitated to possess. For convenience, her various aspects or energies are regarded as separate faculties, but faculties subsumed under reason and derived or abstracted out of reason. These are by no means to be looked upon as extensive determinations or limitations, or as distinct, separable, coordinate, or subjected faculties; but solely as the one faculty reason taken in a limited view, with reference to her particular use, theoretical or practical. In each energy reason herself is present with her whole force and weight. It cannot be urged that an inferior faculty (e. g. receptivity) may exist without reason; for that fact does not concern the argument, which is not addressed to beasts.

§ 44. As a faculty of perception, reason is termed intuition.

As a faculty of apperception, reason is termed consciousness.

As a faculty of cognition ("for the production of unity of phenomena by virtue of rules"), reason is termed understanding.

As a faculty of subsumption, reason is termed judgment. (E. g., that an object stands under a conception of the understanding, is declared by judgment.)

As a faculty of moral judgment, reason is termed conscience. (Declaring that a given act stands under a rule of right and wrong.)

As a faculty of action (in the sensible world), reason is termed will.

As a faculty of appetition, reason is termed desire (not wish).

As a faculty of desire viewed as habitual, reason is termed appetite.

As the faculty appetite plus the consciousness of her own ability by her own act to produce the object sought, reason is termed choice.

As the faculty of appetition when the said consciousness is not present, reason is termed wish.

§ 45. Surely, the reader may say, here is embarrassment of riches! Riches are wholly relative; and not much may embarrass him who earns but a farthing a day. When the inward ground of the determination of the appetite (consequently when the option) depends upon the reason of the subject himself, the faculty reason is termed will. Choice is therefore appetite in respect to the action; will is appetite in respect of the ground of the action; and wish is appetite inactive. These terms are all needed. And appetite itself in a narrower sense may be taken to signify will or choice when the ground of the determination of the choice does not lie within the reason of the subject himself. For man, as an intelligent living in a sensible world, whose reason is a faculty of receptivity, finds by nature his choice affected by foreign stimuli to so great an extent that his reason continually recalls him to herself and incessantly reconquers her just domain only by force of endless struggle.

§ 46. That a particular is included in a general proposi-

tion, is declared by reason. In this aspect, reason is "the faculty of principles," of "synthetical cognitions from conceptions," "for the production of unity of rules under principles." Like as for convenience the whole activity of reason is figured to be divided and subdivided into various faculties, so the term itself REASON is reserved for this her especial and most supreme energy, which is manifested in treating of naked idea, totally dissevered from even the purest forms of sense. Here reason revels in a new and unknown world, so completely her own that if she lose firm hold on herself she can find no more any ground whereon to rest. Here she exalts her throne and reigns unopposed, herein having no longer to fight the world, the flesh, and the devilish oligarchy of passions, in enforcement of her behests. But this her high place she may not dare to abdicate, not for one instant, in favor of any foreign potentate whatever; for in that one instant she will have resigned not alone her ideal sovereignty, but no less her real title to obedience. He who meanly confesses that his own reason is not his very own, what is he?

§ 47. Although neither the existence nor the possibility of ethico-active reason can be known completely apriori, it yet appears upon the most abstract consideration that to cogitate reason solely as theoretical (speculative) and as possessing no practical power, is absurd; and in view of the rational agent it must also be held self-destructive. If reason is a supreme ruler, she must possess a title to obedience, a power to enforce her commands in the phenomenal world: i. e. she must be a faculty. Else might the subject, while theoretically and in general acknowledging obedience to his sovereign, practically and in every individual fact and instance rebel, impelled thereto by the mechanic necessity of his phenomenal character. And while it is true that the causality of reason in the sensible world cannot be known apriori, it may be known apriori that if the possessor of reason is to reside within the world of nature and to be clothed with a phenomenal character, and if he is to take with him thither his supreme faculty reason, that faculty also must possess a real character, a will. Else were she left behind in the world whence he comes, and were no real faculty. But that she is a real faculty may be known by her phenomenal energy of

understanding, through which alone is given her supersensible ideality. And as he who assails the hypothetical real will can scarcely do so except by a general denial of the real character of reason, he may well be left to extricate himself from an argument which turns upon and devours itself and the very ground upon which it stands.

§ 48. Hence it is proper that an apriori deduction of morality from pure reason should begin hypothetically (§§ 12, 17) by assuming the practical power of spontaneous action which is discovered by reason in experience and immediately by her reclaimed and asserted apodictically. No argument which may seem to deny the possibility of this postulated freedom can possibly be acceptable to reason, which MUST (§ 12) thereupon recommence and indefatigably continue her investigation till she has established her idea satisfactorily.

§ 49. As an exhibited science, practical reason is no other than speculative reason (§ 32). Now science, always speculative, does not propound theories to account for possibilities: but from the very first establishes its postulates in order to rest thereon ascertained (aposteriori) facts. It is difficult to apprehend anything more ridiculous than a natural philosopher engaged in the deduction of imaginary facts. The fact must be at least apparent (quasi), before the commonest reason will undertake an investigation whether it is real and well grounded.

How then shall reason in the practical field (wherein she is no less one and the same reason) be required to deduce one knows not what, first casting aside her own apodictic deliverances? And with what face shall he who having first conditionally accepted a natural fact calls then upon reason to account for it, demand that the moralist shall first reject the experienced fact freedom and shall thereafter categorically deduce —— what shall we say?

§ 50. Descending from the exalted realm of reason into the real world, we find the fact morality, a fact which ever-active conscious reason of all grades and in all times persistently claims as her very own. And since she cannot refuse to maintain and establish her very own, she boldly postulates her hypothesis, freedom of will, and deduces thence morality.

To him who objects that the postulated causality conflicts with the necessity of nature, she proves by the critical analysis of her own functions and faculties that no real conflict exists; and him who flatly denies both her (aposteriori) fact and her hypothetic spontaneity (apriori act), she meets upon equal ground and strips of every, even the simplest, principle of understanding.

§ 51. The reconciliation of freedom and necessity is the highest aim of the noblest and purest of philosophers. The argument ought to be familiar to all disciples of IMMANUEL KANT. To reproduce it has not herein been our purpose, but to sketch cursorily the reach and extent of the faculty reason (§ 13). From this point, taking our stand upon the firm fact morality, and challenging any other deduction whatever than that from the spontaneous activity of reason, and resting our claim to a demonstrated science wholly upon rational consciousness of the two states of action ideal and sensible (in other words, upon the conscience of humanity), we concern ourselves merely with so much of speculative reason (commonly termed practical reason) as, knowing nothing of any object of intuition, has nothing to investigate but the power of her own will to make objects real, and that solely in respect of its form and principle. For him who is habituated to abstract thought, it is an easy task to make an apriori deduction of the law of reason, and thence of all laws and morals (i. e. of particular duties). For this purpose, the possibility of practical principles (the possibility of ethico-active reason: §§ 33, 48) must be assumed.

§ 52. It is because of the absolute impossibility of establishing completely apriori the reality of the law's action (§§ 12, 22, 23, 47), that students of philosophy have sometimes apparently abandoned morality as (*qualibet, non quasi*) based upon reason. But reason does not desert her subjects and possessors, though exorcised by the authority of a thousand ages. From the beginning even until now, her humblest agents have been by her assured of the great fact of her vitality in sense, and of the greater fact of the complete harmony of that vitality with the mechanical course of external nature; and this her assurance is so perfect and so apodictic that he who assumes to doubt it thereby falls into an inevitable an-

tinomy with his own faith and conduct (conflict of principles and maxims), patent to all who choose to note.

§ 53. The practical problem which we have to solve is not emancipation from, but control of, nature. In other words, the duty of man is to maintain the supremacy of reason, not to uproot the necessity of nature.

Out of this struggle comes ethic growth, the true virile strength, the highest virtue. And since a mere man cannot arrogantly assume to state the ends of human existence, yet is impelled by the necessity of his rational nature and method to project some end toward which he must aim, he finds the highest satisfaction in declaring as end (for him, so far as he can know) virtue, which consists solely in the ceaseless endeavor to obey the law of reason.

CONSIDERATIONS ON THE DOCTRINE OF A UNIVERSAL SPIRIT.

Written in 1702, and translated from the French of G W. LEIBNITZ, by A. E. KROEGER.

[The following essay will no doubt be welcomed by all who have found it difficult to fully understand Leibnitz's *Monadology*, published in No. 3, vol. i., of this Journal. In the present essay Leibnitz touches and states with the utmost clearness two of the three great principles of his philosophy: the doctrine of Monads — here called particular souls — and the doctrine of a Preestablished Harmony. It will be well to read also, in connection with this article, Leibnitz on the "Active Force of the Body, the Soul, and the Souls of Animals," published in No. 1, vol. ii., of this Journal.—EDITOR.]

Many ingenious persons have believed and believe yet today that there is only one spirit, which is universal and which animates all the universe and all of its parts, each according to its structure and according to the organs which it encounters, just as the same breath of air causes the various pipes of an organ to sound differently; and that thus when an animal has its organs in good order this spirit creates therein the effect of a particular soul, whereas when these organs become corrupt that particular soul turns into nothing, or, so to speak, returns into the ocean of the universal spirit.

Aristotle has appeared to many as having held a somewhat similar opinion, which was renewed by the celebrated Arabian philosopher Averroes. He believed that there was in us an *intellectus agens*, or active intellect, and an *intellectus patiens*, or passive intellect; and that the former, coming from without, was eternal and universal for all; whereas the latter, being particular in each one, vanished at the death of the individual. This doctrine was that of some *Peripatetics*—as Pomponatius, Contarenus, and others—for some two or three centuries; and we meet the traces of it in the late Mr. Naudé, as is shown by his letters and by the *Naudeana* which have since been printed. They entrusted it secretly to their most intimate and able disciples; but were clever enough to say in public, that this doctrine, though certainly true according to philosophy—whereby they meant the philosophy of Aristotle particularly—was nevertheless false according to the faith of the Church. Hence arose the disputes concerning double truth, which was condemned in the last Lateran Council.

I have been told that the Queen Christine had a great inclination in favor of this opinion, and, seeing that Mr. Naudé, who was her librarian, was imbued with it, it is quite likely that he had given her the information which he had gathered concerning these secret doctrines from the celebrated philosophers he had met in Italy. Spinoza, who admits only one single substance, is not far removed from the doctrine of a single universal spirit; and even the New Cartesians, who pretend that God alone acts, establish it without being aware of their doing so. It seems also that Molinos, and some other New Quietists, amongst others a certain author who calls himself Joannes Angelus Silesius, who wrote before Molinos, and whose works have recently been reprinted—nay, even Weigelius before them both—have fallen into this opinion of a sabbath or repose of the souls in God. This is why they believed that the cessation of our particular functions was the highest state of perfection.

It is true that the Peripatetics did not hold this spirit to be altogether universal; for, besides the intelligences which, according to them, animated the stars, they had an intelligence for this lower world, which intelligence performed the function

of an active intellect in the souls of men. They were brought to this doctrine of an immortal universal soul for all men by a false reasoning. For they supposed that an actual infinite multitude were impossible; and that therefore it were not possible that there could be an infinite number of souls,—which, however, they argued, must be possible if such things as particular souls exist. For the world being, according to them, eternal, and the human race likewise eternal, and new souls being always born, there would necessarily be an actual infinity of souls if they all subsisted.

This reasoning appeared to them to be a demonstration. But it was full of false suppositions. For we admit them neither the impossibility of an actual infinity, nor that the human race has existed eternally, nor the generation of new souls; even the Platonists teaching the preexistence of souls, and the Pythagoreans the metempsychosis of souls, holding that a certain determined number of souls always remains and performs its revolutions.

The doctrine of a universal spirit is good in itself, for all those who teach it admit in fact the existence of a Divinity; whether they believe this universal spirit to be supreme, in which case they hold it to be God himself, or whether they believe, like the Cabbalists, that God has created it, which was also the opinion of the Englishman Henry More and of some other new philosophers, particularly of certain chemists, who have believed that there is a universal Archeus or Worldsoul; and some have asserted this Worldsoul to signify that Spirit of the Lord, which floated over the waters, spoken of in Genesis.

But when men go so far as to say that this universal spirit is the only spirit, and that there are no particular souls or spirits, or, at least, that these particular souls ever cease to exist, they pass, as I believe, the boundaries of reason, and advance without any support a doctrine of which we cannot even entertain a distinct conception. Let us examine a little the apparent reasons on which this doctrine might be based, a doctrine that does away with the immortality of the soul, and degrades the human race, or rather all living creatures, from that rank which appertains to them, and which has generally been attributed to them. For it seems to me that

an opinion of this extensive character ought to be proved, and that it is not enough to have simply an imagined notion of it, founded on nothing but a very crippled comparison taken from the air that animates the pipes of an organ.

I have shown so far, that the pretended demonstration of the Peripatetics, who held that there was only one spirit common to all men, is of no force, and based altogether on false suppositions. Spinoza has pretended to demonstrate that there is only one single substance in the world, but his demonstrations are pitiable or non-intelligible. The New Cartesians, finally, who believe that it is God alone who acts, have given very little proof of it; and Malebranche, moreover, seems to admit at least the internal action of the particular or individual spirits.

One of the most apparent reasons that have been alleged against the particular souls is that men have been troubled about their origin. The scholastic philosophers have disputed much on the origin of forms, which they understood to include souls. Opinions have been much divided as to whether there occurred an eduction of the force of matter, as the figure is cut out of the marble; or a translation of souls, a new soul arising out of a preceding one just as a fire is kindled by another fire; or whether the souls existed previously, and merely made themselves known after the generation of the body; or, finally, whether the souls were newly created by God each time there occurred a new generation.

Those who denied particular or individual souls believed that the fact of these disputes relieved them of all difficulty; but that was simply cutting the knot instead of untying it. There is no force whatever in an argument like this: men differ in the explication of a doctrine; hence that whole doctrine is false. This is the way in which skeptics reason, and if it were admissible there would be nothing which might not be repudiated. The experiments of our time lead us to believe that the souls and even the animals have always existed, although in small volume, and that generation is nothing but a species of augmentation.* By this all the dif-

* This sentence expresses in a clear condensed manner the essence of Leibnitz's Monad-doctrine.—*Translator's note.*

ficulties about the generation of forms and souls disappear. This does not imply, however, that we deny God the right to create new souls, or to give a higher degree of perfection to those that have already a bodily existence in nature; we speak here simply of that which is common in nature, without entering into the particular economy of God respecting the souls of men, which may be privileged, being infinitely above the souls of the animals. What, in my opinion, has also much contributed to imbue ingenious persons with the doctrine of one single universal spirit is this, that the vulgar philosophers propounded a doctrine concerning the separate souls, and the functions of the soul independent of the body and its organs, which they could not sufficiently justify. They had great reason to desire to sustain the immortality of the soul as conformable to the divine perfections and to a true morality; but, seeing that in death the organs which we perceive in animals are deranged and finally corrupted, they believed themselves obliged to have recourse to separate souls; that is, to believe, that the soul subsisted without any body, and did not cease its thoughts and functions after having lost the body. In order to prove this better, they moreover endeavored to show that the soul has even in this life abstract thoughts independent of all material notions. But those who rejected this separate state and this independence as contrary to experience and reason, were thus led all the more to believe in the extinction of the particular soul, and the permanency of the one universal spirit.

I have carefully examined this matter, and have shown that there are veritably in the soul some objects of thought or of the intellect which the external senses do not furnish; namely, the soul itself and its functions*— *nihil est in intellectu quod non fuerit in sensu, nisi ipse intellectus;* — and those who uphold the universal spirit will easily agree to this since they distinguish it from matter: but, nevertheless, I find that there is never an abstract thought which is not accompanied by some material images or traces, and I have established a perfect parallelism between that which occurs

* This is precisely Kant's point: that all our knowledge is empirical except the knowledge of knowledge itself; and that, hence, a science of metaphysics is possible only as a science of knowledge.—*Translator's note.*

in the soul and that which happens in the material world,* having demonstrated that the soul with its functions is something altogether distinct from matter, but that nevertheless it is always accompanied by organs which must respond to it, and that this relation is reciprocal and will remain so always.†

Now so far as the complete separation of body and soul is concerned—although I can say nothing in regard to the laws of grace and to what God has ordained concerning the human and particular souls, beyond what the Scriptures say—since these are things we cannot know through reason, depending as they do upon revelation itself and God himself—I see no reason, neither of philosophy nor of religion, which would oblige me to abandon the doctrine of the parallelism of body and soul, and to adopt in its place the doctrine of a complete separation. For why cannot the soul always retain a subtle body, organized according to its condition, which might even resume some day — on resurrection — that part of its visible body which is necessary; since we do accord to the blessed a glorious body, and since the fathers of the Church have accorded a subtle body to the angels?‡

This supposition is, moreover, conformable to the order of nature, as established by experiments; for the observations of very able observers induce us to think that the animals do not commence to exist as the vulgar think, and that the seed-animals or the animated seeds have subsisted ever since the beginning of things, and that, since order and reason demand that that which has existed from the beginning also should have no end, thus just as generation is nothing but the growth of an animal, transformed and developed, so also death can be nothing but the diminution of an animal, transformed and enveloped, and that the animal should remain the same during all these transformations just as the silkworm and the butterfly

* In his doctrine of the Preëstablished Harmony.—*Translator's note.*

† This same point, of the soul being eternally accompanied by bodily organs, is particularly dwelt upon in Fichte's "Facts of Consciousness."—*Translator's note.*

‡ The younger Fichte, and indeed most of the German philosophers of this day, are developing this point of a more subtle body growing in us during life and becoming our habitation after death, with especial force in their writings on psychological as well as anthropological science.—*Translator's note.*

are the same animal. It is well to remark here that nature has the cleverness and kindness to discover to us its secrets in some small examples in order to lead our judgment concerning the rest, everything being corresponding and harmonious. It is this that nature shows in the transformation of the caterpillar and other insects—for the flies also come from maggots—so that we should be led to divine that there are transformations everywhere. The observation of insects has thus also done away with the vulgar opinion that these animals generate each other by their nourishment, without propagation. It is thus also that nature has shown us in the birds an example of the generation of all animals from eggs, which all recent discoveries have compelled us to admit. This is also the result of the experiments made with the microscope, whereby it has been proved that the butterfly is nothing but a development of the caterpillar, but chiefly that the seeds contain already the plant or the animal full-shaped, although transformation and nutrition or growth should afterwards be necessary to make of the seed one of those animals that are perceptible to our ordinary senses. And as the smaller insects generate themselves in this way by the propagation of the species, we must assume the same of those little seed-animals; that is, that they also come themselves from other still smaller seed-animals, and that thus they have never commenced to exist except when the world commenced to exist;—which accords very well with the Bible, which insinuates that the seeds have existed from the beginning.

Nature has given us, in sleep and swoons, examples which ought to make us assume that death is not a cessation of all functions, but solely a suspension of certain preeminently perceptible functions. I have explained in other works an important point, which, not having been sufficiently considered, has led men more easily to adopt the doctrine of the mortality of souls, namely, this point: that a great number of small perceptions, being equal and balancing each other, and having no set-off or anything whereby to distinguish them, are not remarked by us, and that hence we do not remember them. But to conclude from this that in such states the soul is altogether without functions, is just as when the vulgar believe and assert that there is a void, and that there

is nothing wherever there is no matter perceptible; and that the earth is without movement, because its movement, being uniform and without shakes, is not perceptible. We have an infinity of small perceptions that we cannot distinguish; for instance, a great stunning noise, as the murmur of a vast assembly of people, is composed of all the small murmurs of particular persons, which we cannot perceive in part, but whereof we nevertheless have a feeling, for otherwise we should not feel anything. Thus, when an animal is deprived of organs that furnish it with tolerably distinct perceptions, it does not follow that there do not remain in it smaller and more uniform perceptions, nor that it is deprived of all organs and all perceptions. The organs are merely enveloped and reduced to small volume; but the order of nature demands that everything should redevelop itself and return some day in a perceptible state, and that there is in these vicissitudes a certain well-regulated progress which causes things to die and become more perfect. It seems that Democritus himself has seen this resuscitation of animals, for Plotinus attributes to him the teaching of the doctrine of resurrection.

All these considerations show, how not only the particular souls but even the animals subsist, and that there is no reason to believe in an entire extinction of souls, or even an entire destruction of animals; and that hence we need not have recourse to a universal spirit, and thus to deprive nature of its particular and subsisting perfections; to do which would be indeed not sufficiently to consider order and harmony.* There is, moreover, much in the doctrine of a single universal spirit that does not sustain itself, and becomes involved in greater difficulties than the ordinary doctrine.

For instance: it is at once apparent that the comparison of the air that causes various pipes of an organ to sound differently flatters the imagination; but it explains nothing, and rather insinuates the very reverse. For this universal breath of air in the pipes is nothing but a collection of a quantity of

* The eternal existence and immortality of the same animals and animal souls or monads is one of the most important points in Leibnitz's system, and is in point of fact identical with modern doctrines on the conservation of force, cells, &c.—*Translator's note.*

particular breaths of air, since each pipe is filled with its own air, which may pass even from the one pipe into the other; so that this comparison would rather establish particular souls, and favor even the transmigration of souls from one body to another, just as the air changes pipes. Again: if we imagine the universal spirit to be, like an ocean, composed of an infinite quantity of drops, that are detached from it whenever they animate some particular organic body, but reunite themselves in their ocean after the destruction of the organs, we still form a gross and material notion which does not touch the point, and is embarrassed with the same difficulties as the illustration of the breath of air. For as the ocean is a mass of drops, so God would also be an assemblage of all souls, in pretty much the same way as a beehive is an assemblage of those little animals; but, as this hive is not in itself a veritable substance, it is clear that the universal spirit would also not be in itself a veritable Being, and, instead of saying that it is the only spirit, we should say, rather, that it is nothing at all in itself, and that there is nothing in nature but particular souls whereof it is the mass. Besides, these drops, reunited in the ocean of the universal spirit after the destruction of the organs, would be, in fact, souls subsisting separate from matter, and thus we should fall again into what we wanted to avoid; particularly if those drops retained some remnant of their previous state or had still some functions, and could even acquire more sublime functions in this ocean of the divinity or of a universal spirit. On the other hand, if we assume that these souls, reunited in God, are without any proper functions, we fall into an opinion which is contrary to reason and all sound philosophy, namely, that any subsisting being can ever arrive at a state wherein it is without any function or impression. For one thing joined to another cannot but have its particular functions, which, joined to those of the other things, produce as a result the functions of the whole; otherwise the whole would have no functions since its parts had none. Besides, I have shown elsewhere that each being retains perfectly all the impressions it has received, even though these impressions be no longer perceptible separately, having become joined to so

many others. Thus the soul, reunited with the ocean of souls, would remain always the same particular soul it had been, but separated.

This shows how much more reasonable it is, and more conformable to the order of nature, to let particular souls subsist even in the animals and not outside in God, and thus to conserve not only the soul but likewise the animal, as I have explained above and in other writings; and, moreover, thus to let the particular souls remain always in action, that is, in particular functions which suit them and which contribute to the beauty and the order of the universe, instead of reducing them to the sabbath in God of the Quietists, that is, into a state of slothfulness and inutility. For so far as the beatific vision of blessed souls in heaven is concerned, it is quite compatible with the functions of their glorified bodies, which will always remain organic in their manner.

But if anyone should wish to assert that there are no particular souls at all, not even now, when the functions of thought and sensation are achieved by means of our bodily organs, he would be refuted by experience. which teaches us, it seems to me, that we are something in our individual particular self, something which thinks, apperceives, and wills; and that we are distinguished from other individuals, who think and will something else. Otherwise, indeed, we fall into the notion of Spinoza, or of similar authors, who maintain that there is only one single substance—namely, God—who thinks, believes and wills one thing in me, but who also thinks, believes and wills the very contrary in somebody else —an opinion whereof Mr. Bayle has well exposed the ridiculousness in some parts of his Dictionary. On the other hand, if there is nothing in nature but the universal spirit and matter, we must admit that if it is not the universal spirit itself which believes and wills opposite matters in different persons, it must be matter that is different and acts differently; but, if matter act, what is the use of a universal spirit? If matter, however, is nothing but a first passive, or a pure passive, how can we attribute actions to it? It is, therefore, much more reasonable to believe, that, besides God, who is the supreme active, there are a quantity of particular actives, since there are a quantity of particular and opposite actions and

passions that cannot be attributed to one and the same subject, and that these actives are none other than the particular souls.

Moreover, we know that there are degrees in all things. There is an infinity of degrees between any kind of movement you may suppose and perfect rest, between solidity and perfect fluidity without any power of resistance whatever, between God and Nothingness. In the same manner there is also an infinity of degrees between any active you may suppose, and the purely passive. Hence it is not reasonable to admit only a single active, that is, the universal spirit, with only a single passive, that is, matter.

It is further to be considered that my *matter* is not a thing opposed to God, but rather opposed to the limited active,* that is, to souls, or to the form. For God is the Supreme Being opposed to nothingness, from whom matter as well as all forms result, whereas the pure passive is something more than nothingness, being capable of somewhat, whilst no attribute can be attached to the Nothing. Hence we must combine, in our thinking, with every particular portion of matter particular forms, that is, souls and spirits conformable to it.

I do not wish to recur here to a demonstrative argument which I have elsewhere employed, and drawn from the unities or simple things, wherein the particular souls are included, which indispensably obliges us not only to admit particular souls, but to avow likewise that they are by their nature immortal and as indestructible as the universe; and—what is more—that each soul is, in its way, a mirror of the universe without any flaw, containing in itself an order corresponding to that of the universe itself—an order which the souls vary and represent in an infinity of ways, all different from each other and all veritable, thus multiplying, as it were, the universe as many times as possible, and in this way approaching the Divinity as much as possible according to their different degrees, and giving to the universe all the perfection of which it is capable.

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* In Fichte's terminology: the Non-Ego is opposed only to the Ego and not to God.—*Translator's note.*

After this, I do not see what reason or excuse any one can have to oppose to the doctrine of particular souls. Those who do so, agree that that which is within us is an effect of the universal spirit. But the effects of God are subsisting, not to mention that even the modification and effects of His creatures are in some manner durable, and that their impressions merely connect with each other without being thereby annihilated. Hence if, conformably to reason and experience, and as we have shown, the animal, with its more or less distinct perceptions and with certain organs, subsists always, and if thus that effect of God subsists always in those organs, why should it not be permitted to call it soul, and to say that this effect of God is an immaterial and immortal soul which resembles in some manner the universal spirit? especially as this doctrine removes all difficulties, as appears from what I have said here, and in other writings by me on the same subject.*

* Kant, in his *Critic of Pure Reason*, in the *Appendix to the Amphiboly of the Conceptions of Reflection*, enters upon quite an extended criticism of Leibnitz's system—a criticism that has always provoked discussion both as to the occasion and the ground of it. It seems to me that Kant took the occasion from Wolff's system, which at that time prevailed all over Germany, and was universally held to be a systematic representation of Leibnitz's doctrines, and that he was unacquainted with Leibnitz's own writings. Kant's criticism is this, that Leibnitz was, like all previous philosophers, a one-sided thinker; that is, a thinker who is not aware that the two opposite categories of reflection, which can be applied to every subject under consideration, are of equal validity, and that the truth is in taking both these opposite views on every occasion. This is the reason why the criticism occurs as an Appendix to the Amphiboly of the Conceptions of Reflection. Now, it is quite true that Leibnitz never uttered this truth with that clear self-consciousness which gives immortality to Kant's work; but in my opinion it is equally true that Leibnitz was fully aware of it, and became, indeed, aware of it the moment he discovered the Differential Calculus. Kant opposes Leibnitz to Locke, as if Leibnitz had held the one category of reflection only and Locke the other only. So far as Locke is concerned, Kant's criticism is true enough; he, like all English and most other philosophers, clings to his chosen category of reflection, and polemicizes against the opposite category as if that were altogether in the wrong. But any attentive student, by simply reading Leibnitz's correspondence with Clark, can convince himself that Leibnitz takes especial pains to give to the two opposite views equal validity, and protests only against Locke's one-sided assertion or application of a category of reflection. For further proof, I refer to my article on Leibnitz in the *North American Review* for January, 1869, p. 26–27.—*Translator's note.*

FACTS OF CONSCIOUSNESS.

Translated from the German of J. G. FICHTE, by A. E. KROEGER.

PART FIRST.

THE THEORETICAL FACULTY.

CHAPTER II.

CONCERNING INTERNAL PERCEPTION OR REFLECTION.

All our internal perception presupposes, firstly, an activity of the mind whereby it can free itself from its condition of external perception, and hence posit itself both as a knowing of itself as knowledge (that is, of a limitedness of itself through external perception), and as a knowing of itself as a creative principle (that is, of a power in itself to free itself from that limitedness), which activity of the mind is called intellectual contemplation; and, secondly, an activity of the mind whereby it objectivates this its own power and posits it as an independently existing thing, which activity is called intellectual thinking.

A.—Having thus analyzed the facts of consciousness in *external* perception, it seems that we might now, without further preliminaries, proceed to an analysis of *internal* perception, or reflection, as our second chapter.

But since, as it partly is known already and partly is evident at the first glance, this reflection or internal perception is a condition altogether different from—nay, in part, utterly opposed to—that of external perception, it may seem curious to many how such opposite determinations are possible in one and the same consciousness; and hence, before going further, we first ought to answer this question: how is it possible for the life of consciousness to proceed from one of its conditions to its opposite; or, how is it possible for us at all to proceed from our first to a second chapter?

To solve this question, let us consider together, and let me beg you to find in your own minds true the following:

1. I assert that knowledge in its inner form and essence is the *being of freedom*. What freedom is, I assume to be known to you. Now, of this freedom I assert that it exists absolutely; not, as some one might suppose at the first view, as a quality of some other in-itself-existing substance and inhe-

rent in the same, but as an altogether independent being or existence, and that this independent and peculiar being of freedom is knowledge. I assert that this independent being of freedom places itself before itself as knowledge; and that whoever wants to comprehend knowledge in its essence, must think it as such a being of freedom.

Explanatory.—Here already we get a glimpse of an altogether other, higher, and more spiritual being than common materialistic understanding is capable of thinking. That understanding can very well join something like freedom to a substance as its background, which substance, if closely examined, is however always of a material nature; but finds it very hard, nay, if it has been kept on the wrong track for a considerable time, altogether impossible to arise to a comprehension of an independent existence of freedom. To *prove* such a pure being of pure freedom is a matter belonging to the Science of Knowledge; at present I only ask you to consider such a thought as a possible, problematic thinking. Nevertheless, it can be made clear even here, in immediate contemplation, that knowledge may be actually and in fact such a being and expression of freedom. For in my knowledge of the actual object outside of me, how is the object related to me as the knowing? Evidently thus: its being and qualities are not mine, and I am free from both, floating above and altogether indifferent in regard to them.

2. In every *determined* knowledge, that general freedom which exists, and exists as certainly as a knowledge in general is, is limited in some particular manner. In every determined knowledge there is a duplicity melted into a oneness: *freedom*, which makes it a knowledge; and a certain *limitation* or cancelling of this freedom, which makes it a determined knowledge.

3. All change and all alteration of the determinations of the one general knowledge (or of the one general freedom) must, therefore, consist in either the making loose of latent freedom, or the making latent of loose freedom.

4. But further: since this freedom is to be nothing but freedom and knowledge generally, nothing but the being of absolute freedom, such a making latent or loose of freedom can

be achieved solely through freedom itself. Freedom itself is the principle of all its possible determinations; for if we were to assume an outside ground of those determinations, freedom would not be freedom.

5. If freedom is in any respect latent or chained down, it is in the same respect not loose or free, and *vice versa;* and thus it becomes comprehensible, how various moments of the one universal knowledge must dirempt as altogether opposite to each other.

6. Thus we arrive at the idea of a certain limiting and freeing, or of a *Fivefoldness* together with an *Infinity* in consciousness.

B.—1. Let us now apply these principles, first of all in general, to reflection. In external perception, the altogether simple consciousness—which in no manner rises above itself, or reflects upon itself and the life whereof is therefore not in the least more developed than is necessary to constitute it consciousness—is confined to a determined imaging of its sensation. That freedom which it needs, to bear but the form of knowledge, it receives through the objectivating thinking, which lifts consciousness, though confined to a determined imaging, at least beyond its mere being and frees it therefrom. Hence, in this simple consciousness confined and liberated freedom are united. Consciousness is confined to imaging, but liberated from being, which being is for that very reason transferred to an external object; and hence our knowledge begins necessarily with the consciousness of an external object; for it could not begin lower and yet remain knowledge. In this simple consciousness there is freedom merely of being; and this is the lowest and last grade of freedom.

2. Now knowledge is to rise beyond this determined confinedness of external perception through reflection. It was confined to imaging, and hence must make itself free and indifferent in regard to this imaging, just as in external perception it was free and indifferent in regard to being.

Through the being of a determined freedom there always arises a determined knowing. Here we have freedom from

imaging; hence there must arise a knowing of the image as image; whereas in external perception there occurred a knowing merely of the thing. Here it becomes quite clear, as I said before, that a determined consciousness is the being of a determined freedom. For that, in relation to which freedom is free, is always the object of this determined consciousness. Thus in external perception there was freedom solely in regard to being; and hence arose a consciousness of being, and altogether nothing more. In reflection there is freedom in regard to the imaging, and hence to the above consciousness of being there is joined now the consciousness of imaging. In external perception consciousness said simply: the thing is. But in reflection the newly-arisen consciousness says: there is also an image, a representation of the thing. Moreover, since this consciousness is the realized freedom of imaging, knowledge in respect to itself says: I can image or represent that object or not, as I choose.

3. We have here various new creations:

Firstly, there is as the ground of this newly-arisen consciousness of the image a real self-liberating, a self-liberating on the part of the life of knowledge itself. The determined consciousness, here of the image as the being of a determined freedom, is nothing but the result of the tearing itself loose from its chains on the part of free life, is simply the result of this determined higher life-development on the part of freedom itself. That standing and permanent being of freedom, which now is consciousness, is absolutely created through freedom. Hence this act appears even in consciousness as a gathering together and an exertion.

Secondly, there arises here the knowledge of an image as something altogether new. Did not, then, that external perception which preceded the reflection also contain an image, or not? If the life of consciousness is altogether free, as we have seen it to be, that external perception could surely have entered it only through its own freedom, and thus it seems that the image in external perception must also be always recognized as an image created by freedom. How to think such a thought we here lack even expression. But so much we can say, that the image of external perception could not have been created by a freedom of *actual* knowledge, since

actual knowledge presupposes it as its starting-point, and that hence it is proper to say: external perception did contain not an image but a thing.

But this is merely preliminary. Let us now enter upon a more profound description of the freedom, arisen through this new life-development, in its relation to the image.

In external perception we had, *firstly*, a limitation of the external sense through a determined quality; for instance, of a red color. Hence the freedom opposed to it, the liberation from that confinedness, must consist in a power to freely produce such images of qualities; for instance, an image of not only a red color, but also a yellow color, &c.: a free power of imaging, a power of imagination in regard to sensuous qualities. But since an image of a quality is not possible without a previous actual affection through the external sense, and since a good supply is necessary for a free oppositing of many such images, it follows that life must have existed in a condition of mere perception for some length of time in order to be able to rise to such a freedom of imagination.

In external perception we had, *secondly*, a contemplation of extension, and a contemplation of the thing perceived which was confined precisely to this figure, this size, and this location in universal space. Hence the liberation from this sort of confinedness must consist in this, that the imagination, though always confined to extension in general, has a power to freely imagine figure, size, and location.

External perception involved, *finally*, an objectivating thinking. This, while remaining, on the whole, the same—namely, in that the product of imagination is also objectivated or externalized—must be changed so (the limitation of the external sense in general having vanished) that it is posited as the thinking of an object not actual and in fact existing, but merely imagined and freely thought.

Thus the freedom of imagination is actually a real liberation of spiritual life. For, while we wake, our external sense is still determined and affected by that power which as yet is to us unknown; and it is only imagination which lifts us above this affection through the senses, and makes us capable of withdrawing ourselves from its influences by withdrawing our perception and surrendering ourselves exclusively to the

productions of the imagination, thus freely creating an entirely different sequence of time, which has no connection whatever with the time-sequence of sensuous development. In children, during the first years of their lives, this power of abstracting from sensuous impressions doubtless does not exist, and hence also not the power of free imagination. In grown-up persons the strength of this power of abstraction has various grades, according to the standard of their spiritual development. Archimedes was not disturbed in his geometrical constructions by the tumult of a conquered city; but it is a different question whether he would not have been disturbed had a stroke of lightning flashed down near him.

C.—Let us now investigate this still more profoundly by rising from the determined external characteristics of this new freedom to its inner form.

1. In external perception the life of knowledge has causality through its mere being; and, moreover, a determined causality, since causality in general is nothing real, but a mere thought. It is through this having causality that that life of knowledge (the Ego) rises above the object (the non-Ego); for it is not like the object, a dead, permanent being, but a living producing. But in its moments of perception it is confined to this condition of having causality, and, since it cannot generally be confined, it is confined in those moments to a determined causality.

2. The second development of that life, or of the Ego, liberates itself from this confinedness, signifies therefore: the Ego, or life, rises beyond this, having causality through its mere existence, and hence checks this immediate outpouring of its life. But it certainly cannot thereby annihilate all its life. What, then, is it that remains? Evidently a principle which is not a cause through its immediate existence, but which can become such a cause only through the free activity that has arisen through this very new life-development itself. In short, it becomes a principle which, as such, has its separate independent existence, whereas at the first it had existence only as an actual causality. It has, in fact, put its causality, which on the first stage of consciousness was not in its power, now under its newly developed control. Instead of

having as at first a simple existence, it has now attained a double one: a second new being which floats freely over that first simple one; a being which, as its freedom may choose, can be either a permanent self-determined principle, or an unchecked flow of causality.

3. All being of a determined freedom results in a determined knowledge; hence, now that life has made itself a principle, there arises necessarily an immediate consciousness of itself as such a principle. Can this new consciousness be closer characterized?

It certainly has freed itself from a knowledge to which it at first was confined, a knowledge of the object; and through this freeing there has arisen for it a new knowledge, a knowledge of knowledge. But in the same undivided life-moment there has arisen for it a knowledge of itself as a principle, and thus the knowledge of a principle joins together with the knowledge of knowledge into a substantial body of knowledge, a knowing one who is one and the same with the principle; in short, an Ego. I, the knowing, am at the same time the principle which has been liberated from immediate causality. The consciousness, I, starts from a reflection of knowledge and proceeds to that knowledge as a principle; and both become one through their inseparable union in the condition of reflection.

4. Now this Ego, thus first created through the free development of life and entering consciousness, can either remain in this checkedness of its life-development, or surrender itself unto a free constructing of the power of imagination, or surrender itself to external perception.

5. The question now is: whether at this stage of life external perception is in its inner form precisely as it was previously or not. I maintain that it is not precisely so, and everything depends upon getting an insight into this distinction.

a. Through this new development a total change and alteration in the life of consciousness has occurred. Previously this life had causality through its mere being, but now it has no such causality at all; only through its own free act can anything arise in it. It never can even sink back to that previous condition after once having risen above it.

b. Nevertheless the essence of external perception consists precisely in this, that consciousness has causality through its mere being. How, then, can a consciousness, which is no longer a causality, through its mere being perceive externally?

c. Because, although it is no longer confined to that first condition, it can voluntarily surrender itself to it. It can *make* itself to be a consciousness which has causality through its mere being. Such a making or surrendering is well known to every person under the name of *Attention.* The first being, which always remains but does not absorb the being of consciousness, has been joined by a second being which controls the first one. This second being can never be annihilated, but may well surrender itself voluntarily to the first one.

An Illustration.—The perception of a plant by a child before the development of its self-consciousness is distinguished from the attention given by the natural philosopher to the same plant in this manner: the child, if awake, cannot help but see this plant if it falls within its range of vision, since its consciousness is altogether incapable of entertaining another series of observations. But the natural philosopher, even if the plant falls within his sphere of vision, may either see or not see it, as he chooses; for he may fill up the same time of his life with other thoughts. If he chooses to see and observe it, he does so by a free act, and perhaps even by an exertion to tear himself away from his other free thoughts, collecting himself for the purpose of observation: all of which does not occur in the child's mind, since to the child diversion is not possible, as it does not yet possess the diverting power: imagination. Moreover, the child is forced to accept the appearance of the plant as it may chance to present itself, observing particularly parts, which are prominent, by reason of their strength of expression or unusualness, leaving perhaps unnoticed other parts that are not so prominent; whereas the natural philosopher may guide his observation by a certain order, dwelling upon certain parts until he is quite conscious that he has seen them correctly, &c.;—in short, his observation owes its existence as well as its direction to considerate

freedom, whilst in the child both the existence and the direction of its observation result from the child's present standpoint of sensuous development.

6. *Remarks.—a.* I have described external perception as a condition wherein consciousness has causality through its mere existence, and the new character added to it by reflection as a power to check that outflowing of causality, and constitute life a principle through a possible free deed. As an illustration of the first condition, I have pointed to the child in the first moments of its life. In grown-up men such condition should never arise again, nor ever be observed by him in himself. But there does arise a similar condition, in a certain sick state of the mind, which belongs to the province of psychology, and hence does not interest us here as such, but which we may also make use of as an illustration. Namely: a person may accustom himself, particularly if impelled by violent passions, to a free and aimless imagining (or constructing through free imagination as described above) to such an extent that this flow of his imagination begins to flow without any free act of his, altogether of itself, and that thus his sick condition begins to have causality in his imagination through its mere existence, just like the natural condition of the child in its early perception. If a sickness of this kind begins to get such a deep root as to render altogether impossible, in the checking of that flow, a direction of attention to external perception, and an oppositing of external perception to that flow of imagining: it is called *Insanity.*

Now if such a person were to receive sufficient power forever to check that free flow of his imagination, he would then have himself a free principle in regard to that independent and all-devouring power of imagination; just as, in our first description, consciousness rose from its first stage, and made itself a free principle in regard to the independent external perception, which devoured all its being.

b. One more remark on the distinction of free attention from that external perception which forces itself upon the mind. For the latter it is necessary that consciousness should have causality through its mere being. This causality it retains evermore, and it is cancelled by no freedom. The flow

of external perception continues to flow even for the free person, since he also keeps his senses open. It is only upon his consciousness that that causality has no immediate influence; the flow, however it flows, does not take hold of his consciousness necessarily. If it is to take hold of it he must voluntarily surrender himself to it; he must voluntarily put his consciousness into that state of having immediate causality.

If you call external perception x, then in the condition of that perception, x is the centre beginning and end of that whole consciousness; it cannot not be. But in the condition of attention this x has been all through penetrated with freedom; its existence as well as its duration is product of freedom.

D.—Let us now approach an analysis of consciousness as it is in reflection, which we could not possibly undertake before. It has two components:

1. *Contemplation.*—This has been described before as an immediate consciousness of selfhood, of its condition as well as of its faculty. But now we describe it with still greater exactness as follows: contemplation is that kind of knowledge which results immediately from the being of freedom. But in this description we have also a double contemplation, the component parts whereof are just as distinct as they were found to be in the former case: firstly, a contemplation of the condition, and, secondly, a contemplation of the faculty.

a. The contemplation of the condition may be expressed as *a knowledge of knowledge;* a knowledge of a confinedness or limitation of the internal sense through the perception of a determined external object, precisely as the external sense was in external perception limited by the object itself.

b. The contemplation of the faculty may be expressed as *a knowledge of a principle*, beyond all causality. This contemplation or knowledge is (just as we found extension to be in external perception) a contemplation of the faculty of knowledge. But there is this distinction, that whereas in external perception the infinite faculty realized itself actually and had causality, that is, an *actual* infinity, which was pressed together to a totality only through the form of

contemplation, here the principle generally, without any act or causality, is contemplated in its merely *possible*infinity.

Let me ask now: is this consciousness of a principle actually a contemplation? If we look at its *form* we cannot but answer yes, since it is the immediate expression of freedom which lifts itself above causality by its mere being; but if we look at the *substance*, we might fall into doubt. For a principle is an activity that extends beyond each of its possible causalities. Here, therefore, appears a going beyond all possible causalities (which are mere phenomena) as the true characteristic of thinking. We must, therefore, say that, in the contemplation of a principle, the characteristics of contemplation and thinking intimately penetrate each other.

2. *Thinking.*—This has also been described before as an externalizing, and manifests itself here as asserting: "I am; I exist independently — independently even of my knowing myself—now and forever. It is true that I also contemplate myself; but I do not get existence through this contemplation, nor shall I cease to have existence if this contemplation withdraws its breath, for I have an independent and on-itself-reposing existence." Hence there is here a going beyond all possible contemplation, and this going beyond constitutes the real character of thinking. Just as in external perception consciousness did not say, as it ought to have said on the basis of contemplation alone: "I behold such and such," but said, "Such and such a thing is"; so in the present reflection consciousness does not say, "I behold such and such a principle," but, rather, "Such and such a principle is." Now these two or three—as you choose—components of reflection unite here together, just as in external perception, to an organic unity and inseparability. Hence the first named component takes also part in the effect of thinking, and there enters thus into the complete and actual consciousness not only a mere knowledge of knowledge, but moreover an independent being of such a knowledge; hence a knowing mind as the independent bearer of knowledge in all knowledge—at least, in all knowledge of external objects. It is quite evident that this knowing mind is the same in all knowledge

which it originates through freedom. Again: since the principle and the knowing mind get their being through the same one thinking, it is quite evident that this being is also the same; and thus the thought of the Ego is made complete.

Let us ask here, even as above: what sort of thinking is that thinking we have just described? The thinking of the external object was an absolutely unconditioned thinking, a thinking which has existence just as soon as consciousness has existence. But the present thinking is a thinking conditioned by free reflection; hence a second thinking, and probably the second in order.

Furthermore: we observed, in regard to the first thinking, that it would not be proper at all to say: I think this thinking and by means of it the object; but rather: the universal and independent thinking itself thinks the object. So likewise here. The thinking which occurs here first thinks the Ego and gives it its being. For surely the Ego cannot well think before it is, and generate its generator! Hence the Ego is, precisely like the external object, the product of universal thinking, and is given to itself through this thinking just as the external object is given through it.

Hence also I cannot say properly: It is I, the free Ego, who represent this object—for whatsoever in my representation I intermix with my freedom is not objective;—but rather: I am free simply to direct my attention to this object, or to abstract from it.

This is highly important. For, as I assert (and you doubtless have convinced yourselves of the correctness of my assertion by your own observation and contemplation), the Ego—as we for the present call it, and as the ordinary use of language calls it, apart from the Science of Knowledge—posits neither the external object nor itself; but both the external object and itself are posited through the universal and absolute thinking, and this thinking gives to the Ego not only the object but also itself. The free productions of its imagination, the Ego, perhaps, may posit itself. Nevertheless the science of Knowledge has hitherto been generally understood as asserting the very reverse of what I have just now stated. Now it is certainly true that the Science of Knowledge has

said, and will ever say, and says to you now, that the Ego posits absolutely itself as well as in itself the object. But in saying this it does not speak at all of the empirical Ego, but of an Ego which is altogether concealed to ordinary eyes, cannot be found at all within the sphere of facts, and can be recognized only by a rising to the fundamental ground. But this only the Science of Knowledge can justify.

E.—*Remarks.*—1. This is the proper place to state more definitely the peculiar nature of thinking. I have said before that thinking adds no new ingredient whatever to contemplation, but merely gives it another form; elevating it above its flowing, phenomenal nature, and changing it into an independent being. It is thus in the immediate act of original thinking; and the result thereof is, therefore, also independent and permanent, since that thinking is a development and progression of independent life. Now let us suppose that this result of thinking—i. e. the objective being which thinking adds to the object of contemplation—is analyzed just as it is found after that original act of thinking, and we shall find in it a twofoldness, i. e. firstly, a being which has or carries certain qualities, and, secondly, those qualities themselves. And now I would ask anyone to tell me what that being, substance, or bearer of the qualities (accidences) is in-and-for-itself, or whether he has a single word wherewith to characterize it as such being, or whether, if he casts aside this merely formal being, he retains anything else than the qualities. Hence that being or substance (the thing *per se*) is nothing at all in itself, but is merely the accidences in the form of thinking. That bearer is nothing but the eternal being-born by the eternal and universal thinking of the accidences. Now let us suppose further, that I start with my thinking from the substance, and characterize it (as I cannot well do otherwise) through its qualities: how, then, do I name *it* in relation to that which, considered as a mere quality, I name simply *blue*, *round*, &c. I suppose I name it a *blue thing*, a *round thing*, &c., and cannot well name it otherwise. Let us now apply this to the just considered case, wherein knowledge is changed through thinking in reflection into a *knowing one.* "To know"

is a general flowing quality, and expresses an accidental characteristic precisely like blue, round, &c. Now thinking takes hold of this accidentality and raises it into the form of independent being. How, then, must we name that which is discovered in analytical consciousness as the result of such a thinking, and how will it be named by the natural use of language if left to itself? Evidently not a knowledge, but a knowing one, since through thinking there has arisen a substance, and a permanent, firm bearer of all knowledge.

2. It is to be observed, moreover, that we have now discovered two utterly distinct acts of thinking as facts of consciousness. For, we either retain the qualities, simply forming them through thinking, and this is a thinking according to the form of *substantiality*, wherein we have a substance with its accidences; or we proceed altogether beyond the accidences and do not retain them at all, in which case we think a principle, or ground, or the relation of *causality*. Through the first mentioned manner of thinking we have now obtained two substances; *firstly*, the object of external perception, and, *secondly*, the Ego as a *knowing substance*. The second mannner of thinking occurs only in an absolute synthesis of thinking, as we have seen, in which synthesis that thinking, or the Ego, is changed through the first link of thinking into a substance, and through the second link into a principle.

Let us finally observe, that the object of external perception can never become ground or principle, as the Ego is, and as we have explained it to be, but only a *cause* through its mere existence, as will appear hereafter. So far as the Ego is concerned, there is here a twofold relation. In regard to external perception the Ego is purely substance, and by no means principle or ground. The Ego is principle or ground solely in relation to the productions of its inner freedom, and it is only through its being thus a principle that it becomes also the substance of the knowing of these productions. This distinction will be very important hereafter.

THE DEPARTMENTS OF MATHEMATICS, AND THEIR MUTUAL RELATIONS.

By George H. Howison.

I. — HISTORICAL.

The classification of the Mathematical Sciences is a comparatively recent form of exercise for the human intelligence. Indeed, to the ordinary reader, the multitudinous variety of the branches seems to preclude the conception of a harmonious whole. Merely to read the list, following as far as possible the order of study, — Arithmetic, Algebra; Geometry, Trigonometry, Mensuration, Plane Surveying, Navigation; Conic Sections and Analytic Geometry; the Calculus of Finite Differences, the Differential and Integral Calculus, and the Calculus of Variations; Mechanics; Astronomy, Theoretical and Practical; Geodesy, Topographical Surveying, Nautical Astronomy; Engineering, Civil, Military, Naval, and Mechanical; Mining and Architectural Calculations, and Stereotomy; Physico-Mathematics, including Optics, Thermodynamics, Acoustics, and Molecular Mechanics—is not the imagination at once confused by the interminable array of topics? The differences, rather than the resemblances, of the various members of the series strike the attention. We seem to be caught away in an indistinguishable whirl,—the whole appearing as "a mighty maze, and *all* without a plan." Or, if to the thoughtful student there comes to appear a certain coherence and development in the elementary branches of Arithmetic, Algebra, and Geometry, he finds the continuity of his conceptions suddenly broken up when he reaches the province of Analytic Geometry. The interruption only extends itself when he enters the misty region of the Infinitesimal Calculus, and finally leaves him drifting about amid the tantalizing shadows of methods that seem wholly arbitrary. And yet our list, long as it is, gives no hint of how Geometry breaks up and shades away into Plane, and Solid, and Spherical, with episodes of Planimetry and Stereometry, of Stereography and Descriptive Geometry. We have not mentioned

the so-called New Geometry, with its Transversals, its Anharmonic Ratio and Harmonic Conjugates, its Reciprocal Polars and Centres of Harmonic Means; nor the Modern Algebra, with its armament of Discriminants, Determinants, Invariants, and Co-variants; nor yet the Quaternion, just risen into view, with its Fourfold of Impossibles capable of representing every Real, and bringing in a train of Double, Triple, Sextuple, and other Multiple Algebras.

To this manifold diversity, as a source of difficulty, must be added the fact, that the system of mathematics has only unfolded itself by the slow progress of ages, and never assumed its normal proportions till toward the close of the eighteenth century. In fact, its subtlest principles never came to consciousness until the invention of the Higher Calculus in the latter part of the seventeenth century, and another century elapsed before the principles thus revealed had taken consistency and thoroughly reorganized the science. Meanwhile, the great mathematicians were so much absorbed in the exciting business of perfecting the Calculus of Leibnitz and Newton, and in the wonderful physical discoveries to which it led at once, that they had neither time nor inclination for those less fruitful meditations which the philosophy of their science demands.

The result has been, that, although every member of an enlightened community is now familiar with the details of at least one branch of mathematics; and although the elementary study of all its branches known at the time, has in all civilized ages been counted essential to liberal education; still, the co-ordination of its various parts has, until recently, received no proper attention, and may even now be regarded as a matter of controversy. If Berkeley, D'Alembert, Carnot, and Lagrange, had succeeded in reaching the fundamental principles of some of the branches, or in re-establishing a *principle* of continuity which the Calculus of Leibnitz and Newton had seemed to interrupt, the actual co-ordination itself — the comprehension of mathematics as an organized whole — still awaited realization at the beginning of the present century. Comte, the founder of the Positive Philosophy, opens in 1829 his lectures on the *Philosophy of Mathematics* with the following notable statement:*

* *Cours de Philosophie Positive*, tome 1, p. 117.

"Although the science of mathematics is the most ancient and the most perfect of all sciences, the general conception which we should form of it is not yet clearly determined. The definition of the science, and its principal divisions, have hitherto remained vague and uncertain. The plural name by which it is usually designated would of itself be sufficient to indicate the want of unity in its philosophic character, as it is commonly conceived."

And so well founded does John Stuart Mill consider this sweeping assertion, that he declares* that Comte "may truly be said to have *created* the philosophy of the higher mathematics," and that his speculations on this subject "are among the most valuable gifts for which philosophy is indebted to that eminent thinker."

It is to be doubted whether the results of Comte's labors will justify this exalted opinion; but the bare fact of its expression by Mill is sufficient to indicate that the field of mathematical philosophy had been but little cultivated before Comte's day. It is unquestionable that Comte was deeply indebted to some of his immediate predecessors — to Carnot, for instance, and, above all, to Lagrange — but this he has himself declared, with enthusiasm;† and, when all the deductions on this score have been made, he must still be credited with the only serious attempt to set forth comprehensively, in a united system, the Idea of mathematics as a whole, and to carry out in complete detail its logical development into its several branches. To be sure, in 1755, Montucla conceived and partly executed the plan of writing a history of all mathematical science,—a work which he revised and began to republish in 1798, and which, upon his death in the midst of his labors, his friend Lalande brought to completion three years later; but this work, vast in design and compre-

* *System of Logic*, New York, 1846, p. 369. Sixth edition, London, 1865, v. II., p. 153.

† *Cours de Philosophie Positive*, t. 1, p. 118: — "..... les derniers perfectionnements capitaux éprouvés par la science mathématique ont directemente préparé cette importante opération philosophique, en imprimant à principales parties un caractère d'unité qui n'existait pas auparavant; *tel est éminemment et hors de toute comparaison l'esprit des travaux de l'immortel auteur de la 'Théorie des Fonctions' et de la 'Mécanique analytique.'*" Again, p. 244: "..... Carnot présenta enfin *la véritable explication logique directe de la méthode de Leibnitz*.....Carnot a rendu ainsi à la science *un service essentiel, et dont l'importance me semble n'être pas encore suffisamment appréciée.*"

hensive in scope to a degree that will keep it the storehouse for generations of investigators yet to come, falls far short of coördinating the parts of the science,* and labors heavily with its enormous mass of imperfectly digested material. Prompted by his sense of these defects in the work, the mathematician Bossut, while Montucla's revision was coming through the press, published his *Histoire générale des Mathématiques*, a book characterized by thoroughness, brevity, and a clear arrangement based on an attempt to define and classify the science and its branches; but he advanced no farther than to divide mathematics, by the arbitrary rule already in vogue, into *Pure* and *Mixed:* referring to the former class Arithmetic, Geometry, Analysis, and Analytic Geometry; and to the latter, Mechanics, Hydrodynamics, Astronomy, Optics, and Acoustics. So that, while Montucla and Bossut both contributed toward *presenting* the total field of mathematics at a single view, neither of them can be said to have brought it under the control of one all-pervading Thought.

Nor do the labors of Hegel interfere seriously with the peculiar claims of Comte. For, though in 1812 Hegel devoted the whole second chapter of his celebrated *Logik* to the dialectic of Quantity, aiming to show up the transition from Quality to Quantity in the Pure Thinking, and to trace the very Ori-

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* Lest I be thought unjust to the patient and thorough Montucla, I here subjoin his attempt at classification:—

I. *Pure, or Abstract Mathematics:*
  1. Arithmetic = Science of Number;
  2. Geometry = Science of Figured Extension;
    a. Elementary, extending through the Circle,
    b. Transcendental; including the other Curves,
      α. Finite.
      β. Infinitesimal.
  3. Algebra = *the Mediation of Arith. and Geom., and including both* [strangely defined, again, as "Arith. by Signs." and as "Science of *any* Relations—of Magnitude in general"] and subdivided as
    a. Ordinary, dealing with Finites, Solution of Equations, and Theory of Curves.
    b. Infinitesimal = the Diff. and Integ. Calculus.

II. *Mixed, or Physico-Mathematics:*
  1. Mechanics,
  2. Astronomy,
  3. Acoustics,
  4. Optics, etc.

gin, or First Rise, of the conception of Quantity in the process of Intelligence; though he offered an absolute definition of Quantity in its total comprehension, and endeavored to unfold its relations to Space, Time, Motion, and Number; though he entered upon a detailed critique of the Mathematical Infinite, and believed that he had exposed the unphilosophical character of the Infinitesimal Calculus as expounded either by Newton or Leibnitz: yet his treatment of mathematics is, upon the whole, fragmentary; thoroughgoing classification is nowhere attempted in it; and his criticism (even granting that its fundamental definitions are correct, which few perhaps will concede) is at best but germinal, and not applied in detail to subordinate the science.*

So far, then, as concerns the actual grasping together of all the branches under one superintending conception, and attempting to work out their logical rise from it with something like a mastery of their details, it seems to me that Comte's claim to prominence cannot be successfully challenged. It must be remembered, however, that this claim rests on his comprehensiveness of grasp alone, some of the most important principles of his classification having been provided to his hand, as already noticed, by Lagrange mainly, and, in a lesser degree, by Berkeley† and Carnot.‡ If, then, the system which he unfolds is in some points marked by a far-and-wide-reaching insight, and in others is still open to serious criticism, the praise and dispraise should be distributed impartially among its several authors.

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* In this comparison between Comte and Hegel, let me be distinctly understood to refer to *comprehensiveness of formal method* alone: Hegel's labors occupy a different field from Comte's, and an altogether higher—as much higher as Essence is than form.

The only other writers who, so far as I am aware, have entered the field of *general* mathematical philosophy, are J. S. Mill and Professor Bledsoe of the University of Virginia. The latter published in 1868 a *Philosophy of Mathematics*, which however makes no claim to exhaustiveness, dealing mainly with Geometry and the Infinitesimal Method. "The author treats Comte, and Mill's opinion of him, with undisguised contempt, asserting that he "has added not a single idea to those of his predecessors, except a few false ones of his own."

Mill's *Logic* and his *Examination of Sir W. Hamilton's Philosophy* both contain chapters giving general views of mathematics, in the Comtian spirit indeed, but under new and interesting aspects.

† Berkeley's *Works*, v. II., p. 422.

‡ Carnot, *Réflexions sur la Métaphysique du Calcul Infinitésimal.*

The very fact, however, that Comte's system thus reposes, in a certain sense, on the highest combination of mathematical and metaphysical genius that preceded him, renders it proper that any later attempt in the same field should start from a correct estimate of his work. I therefore now proceed to the details of his scheme.

Comte sets out with a critique and estimate of the current definition of mathematics — a definition that is current still. *Mathematics is the science of magnitudes*, he quotes; or, rather, *the science which has for its object the measurement of magnitudes.* Of this definition, which to the general reader, or writer, even at this day is eminently plausible, simple, and lucid, he has not a high opinion. It is "vague and meaningless"; it "has singular need to acquire more precision and depth"; it is a "rude outline"—a "scholastic glimpse." His objection to it seems fatal, however; evidently, it does not raise mathematics above the mechanical art of laying off an assumed unit on the magnitude which we wish to measure; it is silent as to any *indirect* measurement of magnitudes, by deriving their measures from the known or readily determinable measures of others with which they are connected by some known law. And yet measurements of this latter class are the staple of mathematics as it exists,—are the only ground on which the title of *science* (or methodized knowledge) can be claimed for it,—are, in Astronomy, Geodesy, Thermodynamics, Celestial Mechanics, its chief glories. How but by indirect calculation can we ever measure the distances of the planets, or prophesy the existence, place, and orbit, of one before unknown? How otherwise can we measure an arc of the meridian; or determine the mechanical equivalent of heat; or demonstrate the law that the planetary periods vary in the sesquiplicate ratio of the mean distances from the sun?

Conceding, however, that the "scholastic glimpse" is "at bottom just," in virtue of mentioning measurement of *some* kind, Comte works his way up from it by means of such considerations as have just been hinted, and arrives at what he terms a "definition worthy of the importance, the extent, and the difficulty of the science." It is as follows:—"We have now come to define the science of mathematics with exacti-

tude, by assigning as its object the *indirect* measurement of magnitudes, and saying that we constantly propose in it *to determine magnitudes one from another, according to the fixed relations which exist among them.*"*

From this definition, which, compared with the former, surely seems vital and full of meaning, Comte sets forth on the march of classification. He finds, as it implies, that every actual mathematical problem in the natural world involves two great steps: the first, the determination of the "fixed relations" among the magnitudes of natural phenomena; the second, the representation of these relations in an equation, and the final determination of the measurement sought by the numerical solution of that equation. He thus provides for two grand divisions, into one or the other of which all the mathematical sciences must fall: to the first, inasmuch as it deals (as he thinks) with the relations of natural phenomena, and seeks the *laws* (or, as he deems them, the *generalized facts*) of their quantification, he gives the name of *Concrete Mathematics;* to the second, whose business is solely to note and reduce the forms of equations, and to combine numbers, by modes of operation determined by thought alone, and therefore independent of the merely phenomenal world, he gives the name of *Abstract Mathematics.* Of the subdivisions of this Concrete branch, he soon disposes: they are, for the time being, *two*,—Rational Mechanics and Geometry, the latter separating again into the ordinary Pure Geometry (which he calls Synthetic or Special), and Algebraic Geometry (which he names Analytic, or General). He is not sure, however, but he ought to include *Thermology*, or the laws of the quantification of Heat, as a *third* principal subdivision of Concrete Mathemathics, co-ordinate with Rational Mechanics and Geometry. This, because the investigations of Baron Fourier had just then resulted in the establishment of direct equations of heat,—algebraic expressions of the quantitative laws of heat itself, independent of any modified application of me-

* Nous sommes donc parvenu maintenant à définir avec exactitude la science mathématique, en lui assignant pour but, la mesure *indirecte* des grandeurs, et disant qu'on s'y propose constamment de *déterminer les grandeurs les unes par les autres, d'après les relations précises qui existent entr' elles.*"—*Cours de Philosophie Positive*, t. 1, p. 129.

chanics. Thus, he evidently regarded the real number of his subdivisions of Concrete Mathematics as liable to increase at any time by the fortunate rise of physical discoveries; in fact, their number should be potentially without limit, or have at all events no other limit than man's capacity to generalize the phenomena of the physical world. And here we must note, that, in ranking Geometry and Rational Mechanics with a science so clearly empirical as Thermodynamics, Comte proceeded deliberately: it being his conviction that all geometric properties, as well as the laws of motion, were mere generalizations from our experience in the natural world. He did not even except the so-called mathematical axioms from this sweeping opinion—an opinion, by the way, in which he enjoys the entire concurrence of John Stuart Mill.

Having thus closed the discussion of the Concrete Mathematics, Comte proceeds to unfold the subdivisions of the Abstract, whose business, as we have already seen, is actually to *derive* the measurements of required magnitudes, in accordance with the laws imposed by the Concrete; thus he finds that, in this province, every measurement, or *value*, is a *derived* value—is obtained from other *given* values by a series of definite operations, inseparable from our experience of the nature of numbers, and whose combinations vary to express the various *laws* of derivation which the nature of geometric, mechanical, or thermological phenomena may require. In a word, in his second province every required value is, as the mathematicians say, a *function* of some given value or values: it is derived from them by some uniform rule, and must vary whenever they vary. He therefore re-names his second province by the title of Analysis, or the *Calculus of Functions*, and advances to the subdivision of this calculus.

And here his doctrine that all the laws which express themselves in functions are generalized from experience, and that the equations expressing those functional laws must therefore become more and more complex as the scale of the phenomena investigated rises higher and higher in complexity, requires him to recognize two general cases: the first, where the quantifications of phenomena are sufficiently simple to

be put directly into equations; the second, where these are so complex that we cannot reach their own relations, but are obliged to call in the aid of certain relations among the elementary constituents of the phenomena, known in mathematics as the *differentials* of original quantities: between these elements, whose relations we can express, and which are connected by uniform laws of derivation with their primaries, the principal phenomena, we then establish equations, and apply to them a calculus which enables us to pass back and forth from them to their primary phenomena and reciprocally. Thus, his general Calculus breaks up into a *Calculus of Direct Functions*, which he calls Ordinary Analysis; and a *Calculus of Indirect Functions*, which he names Transcendental Analysis.

The Calculus of Direct Functions involves two steps: we have to determine general laws of transformation, or general rules for the solution of equations; and we have to apply these to the specific numerical equation of the problem in hand, or determine the numerical value of its roots. So Comte passes immediately to the familiar branches of *Algebra* and *Arithmetic*. Of the singular confusion in his treatment of these, I shall speak hereafter.

The Calculus of Indirect Functions, in general, also involves two steps, or rather phases of procedure: we may have to pass from the primary phenomenon to the "differential" of it; or, backward, from a given "differential" to the primary phenomenon termed its "integral." And these two phases Comte finds so interwoven in many problems, that he groups them together as one, and waits for their counterpoise in his scheme until he discusses that special case (as he calls it) in the Integral Calculus, where we have to pass back from a differential given under such conditions that its integral is *unknown*, and is supposed to be itself gradually *changing*, in accordance with some mathematical law. Thus he finally subdivides his Transcendental Calculus into the *Differential and Integral Calculus* and the *Calculus of Variations*.

His scheme may be recapitulated as follows, inverting the order of the foregoing outline, as he himself does in the detail of his lectures:

I. ABSTRACT MATHEMATICS = *Analysis*, or *the Calculus*.
  1. ORDINARY ANALYSIS = the Calculus of Direct Functions.
    a. Algebra = the Calculus of Functions *per se*.
    b. Arithmetic = the Calculus of Values.
  2. TRANSCENDENTAL ANALYSIS = the Calculus of Indirect Functions.
    a. Diff. and Integ. Calculus.
    b. Calculus of Variations. } [These lack a coördinating Definition.]

II. CONCRETE MATHEMATICS = Investigation of *Calculable Laws of Phenomena*.
  1. [THERMOLOGY = Math. Laws of Heat.]
  2. RATIONAL MECHANICS = Math. Laws of Physical Motion.
  3. GEOMETRY = Math. Laws of Physical Figures.
    a. Synthetic, or Special { Graphic.....Algebraic. / Descriptive.....Trigonometry.
    b. Analytic, or General { of Two Dimensions. / of Three Dimensions.

II.—CRITICAL.

The strong points of the foregoing scheme are obvious:—it sets out with a definition of the science which commends itself at once, to all who are practically conversant with mathematics, as a comprehensive and striking description of what is actually done there; its definitions of the subordinate departments have the quality, essential to science, of describing their subjects by their object-matter; and, in dividing the science into two great realms, the one occupied with the investigation and proof of those laws which govern the derivation of functions, and the other with the actual derivation (or, as the technical phrase is, the *computation*) of the functions, it is not only again in wide accord with the evident facts, but seems at once to indicate, by its luminous and penetrating principle, the pathway through any remaining intricacies of the subject. Doubtless in mathematics we *are* invariably occupied either in establishing laws which connect the value of one quantity more or less immediately with that of another, or else in actually computing such a derived value in accordance with its connecting law. Thus, in Mechanics, we establish numerical laws of motion or force—laws by which the total motion may be derived from the velocity and time, or the acquired velocity from the time and rate of acceleration, or the force from the resistance overcome; in Geometry, we establish numerical laws of form—laws by which we can more

or less directly find the dimensions of one part of a figure from others that are given; while, as results of these two sciences, we form equations expressing the adjustment of such laws, whether mechanical or geometric, to the conditions of any given problem, and then proceed to transform them by algebraic computation, until at last we bring the value of the required quantity within the direct domain of Arithmetic, computing it by some one of the elementary rules. And when this broad distinction between *creating* and *computing* functions is once made, it seems as if our way were tolerably clear to the correct subdivision of the corresponding departments.

But, if we thus recognize the correctness of Comte's scheme in the leading outlines, it seems to me that we must still decide that it is open to serious criticism in many of its most important details. Even to his definition of mathematics, exact as is its descriptive character, the careful metaphysician will have to object that it lacks, after all, the thoroughly scientific quality, because it fails entirely to connect its system of indirect measurement with the essential nature and properties of quantity in general. There being an elemental conception of intelligence called Quantity, and mathematics having by universal consent some very important relation or other to that conception, it devolves upon a complete philosophy of mathematics to show precisely what that relation is, and the exact dialectic by which the conception arises in the process of intelligence, and unfolds itself into such phases as necessitate that general character of combined law-discovery and calculation which we have seen belongs to mathematics. In fact, a failure either to ascend from the correct general description of what mathematics *does*, through all intervening conceptions, to its more generic expression in terms of the supreme conception Quantity, or else to descend from this conception through the proper logical intervals to the natural development of the definition-by-what-is-done, would seem to shut out the subject from the realm of strict science altogether, and to represent it as being a science only in virtue of being a systematic *art*—an art, that is, whose rules can be stated in methodical order. Now, it appears to be an instinctive conviction that mathematics is, on the contrary, a science in

the severest sense, namely, of a system of intelligence pervaded by a sovereign Idea — a conviction which I shall presently attempt to verify by analysis — and this can never be satisfied by any treatment of the subject which reduces its fundamental character to that of an art, no matter how methodical the art may be. Moreover, even should we have to admit that a *complete* dialectic of mathematics, as a phase in the entire science of Quantity, has thus far not appeared by reason of our inability to define Quantity itself, or to exhibit its Rise out of the Prior Elements of Intelligence, still a philosophy accurate as far as it reaches should be competent to show *why* "measurement" is the problem of mathematics, and *why* it is accomplished by indirection—an indirection, moreover, not arbitrary, but coherent according to a determinate system; it should be competent to show just what "measurement" logically is, by exhibiting how it arises out of the interaction of the logical constituents of Quantity itself, as these are taken-up-again into intelligence. That Comte's definition does not even attempt this, is due to the very nature of the Positive Philosophy: he would unquestionably claim that he had finished his task when he was able to assign a description of mathematics so generic as to include all the facts of its use that come within the range of sensuous experience; but we have only to contrast this result with what a searching intelligence asks, as pointed out above, to appreciate the inherent deficiency of a method whose sole convincing evidence is a generalized experience of the natural world.

And this deficiency displays itself more clearly when we come to estimate the detail of Comte's classification. First of all, granting that mathematics, as a method of measurement by means of functions, does naturally fall into the two departments of *creating* and of *computing* functions, with what propriety is the one called *Concrete*, and the other *Abstract?* Surely not because the laws established in the first are in any sense more specific, or more definitely comprehensible, than the rules of computation applied in the second; surely not because a law of algebraic transformation is any more vague or filmy than a law of Mechanics or a proposition in Geometry; surely not because the rules of computation are of more sweeping generality than the laws of motion or

the properties of figure. The rule that one and one make two is indeed necessary in its truth, and universal in its application, wheresoever we find one unit to combine with another, whether in the world of nature or in the universe of thought; but the law that the total motion is jointly proportional to the velocity and time, is likewise necessary in *its* truth, and universal in *its* application, no matter where we find a uniform motion, be it in the finite world of material bodies or in the infinite world of conceivable atoms; and so, also, the theorem that the square upon the hypotenuse of a right triangle is equal to the sum of the squares upon its other two sides, is necessary in *its* truth, and universal in *its* application, no matter whether we consider any right triangle that may occur on the earth, or any of the infinitude of right triangles that we may choose to imagine. Thus the laws of the creative and computative departments of our general Theory of Functions are alike, and equally, universal and necessary: otherwise, indeed, they would not be *laws:* and they only differ in their subject-elements, the former class being laws of *motion* or of *form*, and the latter laws of *number*. Not on the ground, then, of the ordinary and natural distinction in the meaning of those terms, can we call the one class concrete, and the other abstract: let us, therefore, try the distinction which Comte himself proposes. "The first class," says he,* "should be called *concrete*, since it evidently depends on the character of the phenomena considered, and must necessarily vary when we examine new phenomena; while the second is completely independent of the nature of the objects examined, and is concerned only with the *numerical* relations which they present, for which reason it should be called *abstract*." Now, when we reflect that the entire computative side of mathematics is simply a system of transforming and valuing functions by applying to them the fundamental properties of number, Comte's last statement is seen to ignore the manifest fact that his second class is concerned with laws quite as much as his first—with laws, too, that quite as much depend upon the character of the "phenomenon" *number* as do those of Mechanics and Geometry upon the nature of *mo-*

* *Cours de Philosophie Positive*, tome 1, p. 132.

tion and *form*. Moreover, the generic laws of motion and form do *not* vary with the phenomenon considered—unless we mean by this statement, that the laws change when we pass from the province of motion to the province of form, or that the laws of uniform motion differ from those of accelerated motion, and those of rectilinear figures from those of curvilinear; and even when we recognize this last obvious fact, and contrast the variety which the laws of motion and form present, with the almost absolute uniformity of the laws of number,* the truth is not expressed by saying that the laws of number are *abstract* as compared with those of motion and form, but by saying that they are more *simple* and *uniform*. In short, Comte's statement sums itself up in the clear proposition, that the laws of motion are one thing, the laws of form another, and the laws of number still another: which can hardly be gainsaid. But the only warrant that this can give for calling the laws of motion and form *concrete*, and those of number *abstract*, lies in a somewhat plausible play upon words: we may fancy that we cannot conceive of the former except as *mixed up* (*concretas*) with the perpetual flux of physical phenomena, whereas the latter are readily and usually conceived of as entirely *withdrawn* (*abstractas*)† from such a commixture. The plausibility, however, is superficial and deceptive: for, as above shown, the laws of motion and form are as completely separable in thought from the phenomena in which they appear to our senses as are those of number, both being universal and necessary; and, on the other hand, were it true that we only reach the laws of motion and form by generalizing upon the facts observed by the senses, consistency would require us to maintain that all the laws of number, even the fundamental axioms, are reached in the same way;‡ for nothing is more obvious than

* The continuity of the laws of number is only broken by the distinction between *commensurable* and *incommensurable* numbers.

† Thus, we have in our common Arithmetics the time-honored title of *abstract numbers*. The phrase is inaccurate, but nevertheless serves well enough to call up the important truth, that numbers may be taken both abstractly and in the concrete.

‡ This is what Mill actually does maintain. See his *System of Logic*, v. i., p. 260.

that numbers and their combinations are everywhere existent in the natural world, and are therefore a perpetual element of our daily experience.

It thus appears that the attempt to distinguish as *concrete* the establishment of laws of functional derivation, and as *abstract* the application of the properties of number to the actual derivation of functions, is essentially unscientific. The distinction breaks up under the light of reflection: what is called concrete may just as well be called abstract; what Comte calls abstract may, upon his method, just as well be called concrete; or, rather, *should* be called so, thus convicting him of inconsistency in the use of his own principles. And such must of necessity be the result of every attempt to set up as categories of science the Abstract and the Concrete —those twin Phantoms which reflection conjures up to aid it in its progress toward the comprehension of real existence, but which make no part of the abiding truth of that existence, into whose living completeness they are both absorbed.

The unscientific character of Comte's distinction of mathematics into Abstract and Concrete, shows itself in a second defect of his scheme, namely, in the subdivision of the Law-forming department into Geometry, Mechanics, and Thermology, with the implication that the subdivision is in a sort of suspense, awaiting an indefinite expansion by the discovery of new quantifying "laws" in the departments of Physics. The notion of a "suspended" science is of course self-contradictory; yet into this contradiction Comte directly falls, in conceiving of the laws of form and motion as having a "concrete" character merely: that is, as founded solely upon Induction, and therefore of course inviting into their own rank the results of induction known as the "laws of heat." For, in consistency, not only the laws of heat, but those of light, and sound, and color, and magnetism, and electricity, and chemical affinity, and vital force—the laws, in short, of the whole circuit of the Correlation of Forces—should be advanced into the same coördination. In fact, there should be no limit to the number of sciences coördinate with Geometry and Mechanics, except the limit fixed by the possible number of physical phenomena and the power of the human mind to discover their "laws." And since physical phenomena exist of necessity in an infi

nite progression, the possible subdivisions of "Concrete" mathematics should be infinite in number.

This defect in Comte's classification appears to arise from his failing to distinguish clearly between the *essential nature* of a mathematical law, and the *application* of it to the material world; a failure very liable to occur in the attempt to put Mechanics into its proper place in the entire scheme of mathematics, as the treatises upon that branch do not as yet dissever the laws of rest and motion from the equilibrium and motion of *bodies;* so that, as usually discussed, it is more an applied mathematics than a speculative. Nevertheless, the distinction exists; and it is only under the aspect of their *essential nature* that the laws either of form or of motion can have any place in the *science* of mathematics; moreover, the so-called "laws" of heat and light, and so on, are only specific cases of universal laws of uniform or accelerated motion. It is the more remarkable that Comte should insist upon confounding these two aspects of mathematical law, as he has elsewhere clearly discriminated between a science and its application,* and as it must be on the ground of such a distinction that he refuses a place in his scheme—as he does—to Optics, to Acoustics, and above all to Astronomy, which, in its phase of Celestial Mechanics, is assuredly the crowning glory of applied mathematics. On the other hand, had he held consistently to the distinction between science and its application, Applied Mechanics and Thermology would have fallen into their proper place in line with Astronomy, Optics, and Acoustics; and the prospectively endless succession of coördinate branches would have been a legitimate result; for, as Bacon† long ago remarked, "The greater increase Physics receives, and the more new principles it develops, the more will Mathematics be called into new applications, and the more numerous will the *Mixed Mathematics* become."

There is one other point in Comte's treatment of his "Concrete" subdivision which seems to me defective: I mean his discrimination between Pure and Analytic Geometry as "Special" and "General." After very justly criticising the

* *Cours de Philosophie Positive*, tome 1, p. 345.

† *De Aug. Scient.*, lib. iii., cap. 6.

common practice of calling the former *Synthetic* and the latter *Analytic*, he puts his own distinction upon the following ground:—The ancient Geometry can discuss no problem of form or dimension except by taking up all the different figures, rectilinear and curvilinear, one by one, and obtaining a separate solution for each order of figure, triangle, circle, conic, spiral, or whatever else it may be, the *method* of solution having to be invented *new* for every new figure taken up; whereas, in the Geometry of Descartes, problems of contact, of curvature, of asymptotic approach, of singularity in flexure, of the length of the subtangent and subnormal, of rectification, of quadrature, of cubature, are solved in formulæ of absolute generality, which apply to all figures alike. Now this statement, which the mathematical reader will at once recognize as entirely correct, seems at first sight to justify Comte's nomenclature completely; but a closer examination will convince us that it cannot serve to separate Geometry into its scientific provinces, inasmuch as it after all misses the point of *characteristic* difference between Analytic Geometry and Pure. The *method* of Analytic Geometry is undoubtedly more general than that of Pure Geometry; but so is the method of Algebra more general than that of Arithmetic; and yet, as Comte himself has noticed,* it would now be a grave error to define Algebra, with Newton, as a *Universal Arithmetic*, thus setting up Arithmetic as the Special Calculus, and Algebra as the General Calculus. The truth is, that having set out to find the departments of mathematics according to their *object-matter*, it is a formal blunder to introduce a new principle of division, and distinguish between the *methods* according to which a given object-matter is discussed. On the contrary, to raise the so-called Pure and Analytic geometries into sub-departments of Geometry-in-general, we must show that they deal with distinct provinces of its whole object-matter; and, more specifically, admitting that its whole object-matter is *the functional laws which connect the several parts of figures with each other*, we must show that Pure Geometry deals with one class, or else rank, of those functional laws, and Analytic Geometry with another. For this exposi-

* *Cours de Philosophie Positive*, tome 1. p. 177.

tion, we are not ready at this stage of our discussion; but, at a later one, it will appear that Pure Geometry deals with those functional laws which determine *magnitude*, and Analytic Geometry with those which determine *form*.

Passing over, now, to Comte's treatment of the Computative side of mathematics, the chief matter for criticism is his mode of distinguishing the Higher Calculus from the Lower. By defining the former as the *Calculus of Indirect Functions*, and the latter as the *Calculus of Direct Functions*, he grounds his distinction not on the nature of functions as such, but on that of the phenomena in whose "laws" he supposes the functions to originate: for by a *direct* function he means* a quantifying law connecting the parts of phenomena themselves; and by an *indirect* one, a law connecting not the parts of the primary phenomena, but certain auxiliary quantities, united with the primaries by uniform laws of derivation, and substituted for them because *their* laws are too complex to be put into equations directly. Now this mode of distinction, which at first sight seems to gain support from Lagrange's great generic insight of regarding the auxiliary infinitesimals of the Higher Calculus as algebraic DERIVATIVES of the primary functions, has in it, beyond question, this superficial plausibility: it does appear to lift the mind contemplating the "differentials," which play so essential a part in the Higher Calculus, into a field of view which surprises by its unexpected and apparently endless extent; for we seem to see these obscure analytical elements suddenly shining in the wide horizon of the conception of "*auxiliary quantities derivable from their primitives according to any law whatever*," and the present Calculus of Differentials, vast as we thought its generality when compared with ordinary Algebra, appears in the yet grander rôle of one specific method out of an infinite system of possible methods for deriving auxiliary functions. It must be observed, however, that all this splendor of generality is due to Lagrange's conception that differentials are DERIVATIVES, and not to Comte's transcription of that great theme into the notion of Indirection; so that it has nothing to do with viewing differentials as capable of simpler

* *Cours de Philosophie Positive*, tome 1, pp. 189, 194.

relations than the primitive phenomena, except in so far as this notion of "Indirect Functions" borrows the idea of derivation itself. In fact, the formation of functional relations between differentials on the ground of their superior simplicity, belongs not to the *theory* of the Higher Calculus, but to certain of its *applications;* for instance, to rectifications, quadratures, cubatures, and the determination of maxima and minima; in which we certainly do establish some very obvious differential equations for the express purpose of passing from them to integral equations we could not readily obtain by considering the integrals themselves. It is not the procedure of science, however, to define a theory by its applications, particularly by a portion of them. But, above all, this resting the distinction between the two departments of analysis upon the difference-in-complexity of the phenomena to which they are applied, to say nothing of its resemblance to the boy's subdivision of Arithmetic into "*Arithmetic with Easy Sums*" and "*Arithmetic with Hard Sums*," is unscientific because it carries over considerations from Comte's "concrete" field to subdivide his "abstract" one; whereas, if the latter is a real province of mathematics (and, in so far as it is characterized by the idea of *computation*, it certainly *is* a real province), it ought to subdivide *itself*, and *by principles involved in its own idea.* In short, if Analysis *is* the Calculus of Functions, and if there *is* a difference in object-matter between the Lower Calculus and the Higher, that difference must be sought in the nature of the *different orders of computation* which are employed in the two.

And, at this juncture, the root of Comte's inaccuracy comes plainly to the surface: his error grows naturally out of a vagueness in his conception of Analysis itself, and a consequent confusion in his view of the relations between Algebra and Arithmetic. At first, he correctly ranks Arithmetic and Algebra (using this term, in its widest sense, as equivalent to *Analysis*) as the two coördinate branches of the general Calculus (or Science of Computation), defining the former as the *Calculus of Values*, and the latter as the *Calculus of Functions.* But this inexact characterization of General Algebra presently leads him to bring back Arithmetic (which he sees may also be regarded as a calculus of functions) into the prov-

ince of this same Analysis; and he ends at last with the singular contradiction of coördinating Arithmetic with Algebra taken in its ordinary and very restricted sense of a *method for solving equations*, presenting the two as subdivisions of his "Ordinary Analysis" as distinguished from his "Transcendental." This confusion is heightened by the fact that he had, at the outset, in a single passage,* undertaken to explain his phrase *Calculus of Functions* by defining General Algebra as the science having for its object "the transformation of *implicit* functions into equivalent *explicit* ones": a definition which, so far from reaching the comprehension of Algebra as inclusive of all Analysis, is in fact the exact description of *ordinary* Algebra; that is, of the Lower Calculus as distinguished from the Higher. But had he held to the road which leads logically from his starting-point, he would have continued to coördinate Arithmetic with Algebra in its most extended sense; he would thus have recognized that the computation of functions involves their *transformation* and their *evaluation*, and might have defined General Algebra (or Analysis) as the *Calculus of the* FORMS *of Functions*, and Arithmetic as the *Calculus of the* VALUES *of Functions*. And having thus arrived at *transformation* as the essence of all analysis, his path would have been clear to a real discrimination between the Lower and the Higher Calculus, on the ground that they involve two distinct *orders of transformation*.

I need not more than advert to the additional fact, that this confusion as to the nature of General Algebra accounts for Comte's imperfect elucidation of the relation borne by the Calculus of Variations to the rest of the science. Lacking the conception of Analysis as a general method of transformations, he contrasts the Calculus of Variations with the ordinary Integral Calculus; or, rather, treats it as a modified form of that Calculus; whereas, in the light of the idea that functions are *laws* or *forms* of derivation, it ought to be contrasted with the whole calculus of which the Differential and the Integral are the branches, and defined as the *Higher Calculus of* VARIABLE *Functions*, while they in their correla-

* *Cours de Philosophie Positive*, tome 1, p. 187.

tion are defined as the *Higher Calculus of* CONSTANT *Functions.*

With these considerations, we may now leave the labors of Comte, and pass on to essay the actual construction of the System of Mathematics, as it evolves itself in the general procedure of Intelligence.

III.—CONSTRUCTIVE.

We start, then, with the truism that mathematics is a *form of intelligence;* and it is our first business to find the definition of it, by discovering what it is that intelligence is there endeavoring to do.*

And, without venturing to attempt the definition of QUANTITY, let us avail ourselves of our familiarity with that concept as an inseparable factor in all our intelligence; let us also take advantage of our experience concerning the use of mathematics; and we shall be warranted in the provisional statement that *mathematics is that form of intelligence in which we bring the objects of the phenomenal world under the control of the conception of quantity.*

This is vague enough, to be sure; and, were we to stop at this, we should have no real definition. But it is the beginning, from a point commanding universal assent; and, even in this its empty abstractness, it has this immediate advantage:—we see at once that the System of Mathematics must subdivide itself as *Science* and as *Art;* since, as form of intelligence, it must be science, and, as applied to the phenomenal world, it must be art. We thus take a long step toward clearing up the confusion of its multitudinous branches, by resolving the whole into two grand divisions of SPECULATIVE and APPLIED mathematics.

Thus, under the head of Applied Mathematics (the relation among whose subdivisions is determined immediately by the nature of the phenomena with which they deal, and therefore calls for no dialectic), we at one dispose of the greater part

* In thus setting out to fix the system of mathematics by its *motive*, we only obey the controlling principle of all the sciences; since they all, as forms, or *phases*, of intelligence, must have their *raison d'être* in the aims of intelligence—except, indeed, the Objective Logic, which, as the Science of Intelligence itself, is its own *raison d'être.*

of the long list with which we perplexed ourselves near the opening of this article. We may remove, then, as requiring no further discussion: Astronomy, with its subdivisions, Celestial Mechanics and the Calculus of Observations; Surveying—Geodesic, Topographic and Mensurative; Navigation—Celesto-spherical and Plane; Acoustics; Optics; Thermodynamics; Molecular Mechanics; Physico-Mechanics, branching into Statics and Dynamics, Hydrostatics and Hydrodynamics; and Practical Mechanics, including Architecture (with its subsidiary art of Stereotomy) and Engineering in its various branches, Civil, Military, Naval, and Mechanical; not forgetting that these last involve the mathematical principles of Mining, Bridging, Road-building, Fortification, Gunnery, Ship-modelling and Armoring, and of all the parts of Mill-work, whether motors or machines. Any further grouping or defining which these branches may call for, in order to render clear their mutual relations, will sufficiently appear in the tabulated scheme at the close of this article; so that the sketch of this grand division of mathematics may be completed by merely recalling the fact (noticed on a preceding page), that the number of its subdivisions is potentially unlimited, and will continue to increase with man's increasing knowledge of the natural world.

Turning now to Speculative Mathematics, we have in this side of our subject the essence of our whole problem of classification; for the comprehension of a form of intelligence as *science*, is the necessary and sufficient condition of its definition and subdivision. If, then, we can reach the comprehension of Speculative Mathematics—that is, if we can trace in *that* the exact procedure which intelligence makes under its ruling concept Quantity—we shall be able to replace the emptiness of our provisional description by a full and living definition. Let us, then, attempt the exposition of the mind's continuous descent from the concept of Quantity to a science of Mathematics.

A. *Process of finding the Thought-Constituents of Mathematics.*

SPACE and TIME—these are the logical *sine quâ non*—the necessary thought-element—of all phenomenal existences; just as the air is the indispensable matter-element of our

vital existence. Into these as such thought-element, therefore, must intelligence project its concept Quantity, when it seeks to dissolve the phenomenal world by means of that. But Space and Time react upon the projected concept, so that it is itself dissolved; only to rise, however, into a threefold power for intelligence, as Extension, Motion, and Number; for these three "modes" of quantity are simply the thought-phases into which it breaks up under the reaction of Space and Time. Quantity projected into Space considered aloof from Time, is *magnitude;* projected into Time considered aloof from Space, it is *duration;* and these two abstractions disappear in the wider one of EXTENSION, which may therefore be defined as *Quantity projected upon the abstract severance of Space from Time.* But if we let go of our abstraction, and consider Space and Time in their *real* aspect of *coëxistence,* the concepts magnitude and duration flow-together and annul-each-other in the idea of MOTION, which we may accordingly define as *Quantity projected upon the concrete interflow of Space and Time.* Extension and Motion, however, only present Space and Time in their phase of *infinite continuity,*— a thought with which the Conceptive Understanding is entirely unable to cope. To that side of intelligence, nothing but the *finite* — the *externally* limited — is comprehensible; and yet, since it *is* a side of intelligence, its protest against Quantity as pure Continuity must have a hearing. Thus, when this Conceptive Understanding "sinks exhausted" before the thought of unbeginning and unending extension, or of unarising and unceasing motion, and is on the point of rejecting both as meaningless abstractions, the Thinking Reason comes to its rescue: it negates the continuity of Space, and posits the *point;* it negates the continuity of Time, and posits the *instant;* in the infinite of Space, it finds a *Here* and a *There;* in the infinite of Time, a *Then*, a *Now*, and a *Hereafter.* Between these, as limits, the whole mystery of Extension is resolved for the Understanding into a *limited extent* on the one hand, or a *period elapsed* on the other; the whole mystery of Motion, into a *distance traversed in a given time.* Thus Quantity descends into finite terms; and, what is more to the purpose, in doing so passes to its final and most intelligible phase. For when in Infinite Continuity we set up

limits, then, in so far as it is Extension, its unity breaks up into *multitude;* and, in so far as it is Motion, it breaks up into *succession;* but the union of these two constitutes the idea of NUMBER, which may consequently be defined as *Quantity projected upon the annulled-continuity of Space and Time.*

Such is the genesis of Extension, Motion, and Number,—the three elements of thought in whose correlation, as we shall presently see, mathematics finds at once its occasion and the ground of its method.

B. *Construction of the Fundamental Problem of Mathematics.*

NUMBER, from the manner of its logical origin, as that has just been unfolded, is seen to be the triumph of the Conceptive Understanding over the primordial chaos of Space and Time; or, more truly, the reconciliation of the Understanding, which can comprehend nothing but finites, to the infinite continuity of Quantity as seized by the Reason in pure Extension or Motion. It is the instrument for completely simplifying these less determinate phases of Quantity, from which the Understanding naturally revolts as from a bewildering mystery; for, as Number arises out of dissolving Extension and Motion into successive *parts*, it becomes at once their interpreter to the understanding, by presenting all *periods elapsed*, all *limited extents*, and all *distances traversed in given times*, as aggregates of equal parts as small as we please and therefore as easily conceivable. Hence there arises in the mind a persistent tendency to convert all forms of Extension and Motion into Number; and the three are so correlated that this tendency is readily satisfied, and becomes the occasion of that systematic contrivance for effecting this conversion which we call mathematics. Thus it appears that the Fundamental Problem of that science, stated in its universal form, is this: *To pass at will from the mental province of Extension or of Motion to that of Number.*

The general, abstract solution of this problem is exceedingly simple, being in fact given in the very process by which Number arises: we have only to resort to the easy expedient of comparing Extension or Motion with any one of their equal parts, and then considering the "ratio" of the whole to this

arbitrary unit. And here, in our universal language, "measurement" translates itself into "converting extension or motion into number": we see what "measurement" logically *is*, and why it is the problem *par excellence* of mathematics. It remains to discover why it must be solved by "indirection"; that is, by deriving the numerical equivalents of extensions or motions through a complex *system* of relations, instead of by directly applying the unit-part to the measurable whole.

c. *Origin of System in the Procedure of Mathematics.*

The path which leads us to the discovery last mentioned is not long; we perceive, indeed, with but brief reflection, that Space and Time baffle the attempt to apply the expedient of the direct unit as easily as they repel that of bringing their pure continuity within the comprehension of the imagination. In fact, it is the persistence of that infinite continuity, despite its apparent cancellation by arbitrary limits, which effectually annuls the petty device of our measuring unit: in the infinite possibilities of Space and Time, extents, durations, and motions run on forever, and pass beyond our reach when we attempt to lay our measure upon them. Could we actually annul Time, we might actually measure Space by following its successive extensions; could we actually annul Space, we might follow out the successive epochs of Time; but, fortunately, we can really do neither: the two elements *coëxist* in the real world—the actual application of the unit to the whole is, as a universal or even general procedure, an impossibility, and we are forced to abandon this conceptual method and take refuge in the mightier powers of the systematizing Reason. For though Space and Time baffle us when we attack them from the side of our finite existence, they yield at once when we bring against them the idea of Self-Relation. Self-related points we can posit in space; self-related instants, in Time; and out of these arise, on the one hand, Form and Figured Extension, and Rate (or Velocity) of Motion, on the other. To determine the form, or figure, of an extension, is necessarily to determine the magnitude of all parts that can be set up within or about it in accordance with given conditions of position; to determine the rate of a movement, is necessarily to determine the distances traversed by

it in known times, or the times due to known distances. And if, armed with these new resources, we turn to the idea of Number, and observe *its* inseparable properties—how it exists in a ceaseless flow and re-flow; how we can count forward by unity from zero to any number conceivable, or backward by unity from any number to zero again (*addition* and *subtraction*); how, by seizing as unity any number of units, we can sweep by this greater stride to any desired other number (*multiplication*); how, by reversing this process, we can lessen as much as we please the number of backward steps from any number to the original zero (*division*); how we can count any number, taken as unit, as many times as there are units in itself, repeating this as often as we please, (*involution*); how, finally, we can undo *this* process as well as the simpler ones (*evolution*)—we see that all these so-called "operations" of addition, subtraction, multiplication, division, involution, and evolution, also result from the idea of Self-Relation*; the mutual adaptation of the processes of Number, and the interdependences of Figured Extension or Rated Motion, present themselves clearly to our anticipation; so that we return again to the provinces of Form and Movement, in the conviction that the magnitudes of the parts of figures, and the quanta of the constituents of motions, are connected with each other by discoverable *numerical* laws; in other words, that these magnitudes, and these quanta of motion, *grow out of each other at rates expressible by the very "operations" constituting the essential properties of numbers.* And this anticipative conviction the contents of Mechanics and Geometry completely confirm; for all the theorems of Motion and of Figure are, either directly or indirectly, just such statements of the numerical combinations that have to be made of constituent motion-elements or magnitudes, in order to produce certain derivative motion-elements or magnitudes. Thus it is that the mind, seeking to solve the problem of converting Extension and Motion into Number, finds itself possessed of a vast and coherent *system* for effecting the solu-

* Numeration is possible only by this:—that the One, going-out-of-itself into the Many, shall again return-into-itself as the One (of Quality) which brings the Many into being Successive Units: we cannot count the *mere* Many. Thus, the properties of Number arise out of the system of the Self-related Unity.

tion,—a system of *laws connecting change with change*,*—a system for *deriving* values from given ones of any nature by the very modes of combination which constitute the laws of Number itself, and by which numbers are themselves derived from each other.

D. *Definition of Mathematics considered as Science.*

We have now reached the point from which the definition of mathematics, in its aspect of science, comes clearly into view. It may be stated as follows:

Mathematics is the science of converting Figured Extension and Rated Motion into Number, in accordance with the system of numerical laws which connect the parts of figures and the elements of motions with one another.

Or, inasmuch as these numerical laws of mutual dependence constitute the mentioned parts and elements *functions* of each other, we may state the definition more concisely as follows:

Mathematics is the science of the functional laws and transformations which enable us to convert Figured Extension and Rated Motion into Number.

E. *Subdivision of the Science, and Coördination of its Parts.*

The science which actually succeeds in passing over from Extension and Motion to specific Number, and that, too, by means of a System of Functions, will of course involve two grand movements: the investigation of the numerical laws which connect the parts of figures and the constituents of motions with each other; and the working out, from the relations given by these laws, of the actual numerical values of the magnitudes or motion-elements sought. Thus the entire field of mathematics divides itself into two main provinces: the one, a Body of Doctrine concerning the numerical combinations which have to be made of given parts of figures, or given parts in a moving system, in order to obtain required parts; the other, a Theory of Operations by which the doctrines of the first may be represented, transformed, and finally

* Sir W. R. Hamilton, in preface to his *Algebra as the Science of Pure Time*. See the *Transactions of the Royal Irish Academy*, vol. xvii.

evaluated by means of the processes which we call the essential properties of Number. It is our business, in the first, to *create* the System of Functions; in the second, to *transform* and *evaluate* its members when created. In short, mathematics consists of (I.) a MATHESIS, and (II.) a CALCULUS.

I. The MATHESIS of mathematics, again, has two subordinate provinces, *Geometry* and *Mechanics*, depending on whether the functions created are laws connecting the parts of *Figures*, or the constituents of *Movements*. And these two* are *all*, whether actual or possible; inasmuch as, excluding Number, Extension and Motion exhaust the possible modes in which Space and Time can react upon Quantity, as has already appeared at an earlier stage of the present discussion.

1. If we define *Geometry* as the *Science of Figure*, or, more exactly, as the *science whose object is to determine what numerical combinations connect the parts of figures with each other*, we shall not advance far in the detail of verifying the definition by the actual content of the science, without noticing that the theorems of Pure Geometry present two quite distinct classes: by far the greater part of them certainly do connect *magnitudes* by numerical relations which enable us to find one when others are given; but another part, at least equally striking, if much less numerous, seem to be altogether concerned with relations of *position*, that is, with *form:* such are the theorems that *Any three points are always in the same plane;* that *Any three points, not in a straight line, are on the circumference of a fixed circle;* that *A point whose distances from a fixed point and a fixed straight line are in a constant ratio is on the periphery of a conic;* etc. In pure geometry, these do not seem to express relations of magni-

* Should the curious reader here ask—What has become of *Duration*, that only *Magnitude*, as befunctioned by Figure, is taken to cover the whole science of Extension?—I reply that the quantification of Time is so simple as to make no science, its whole mathesis consisting in the law which connects a *period elapsed* with its limiting *dates*. Thus, its sole function is $P = D - D'$; whence, $D = D' + P$; or, $D' = D - P$.

I submit this answer with reserve, however; remembering that the illustrious HAMILTON has attempted the construction of a *Science of Pure Time*, and its absolute identification with the whole of *Algebra*, taken in the widest sense. See the *Transac. Royal Irish Acad.*, vol. xvii.

tude, nor to yield any functions by which magnitudes can be calculated. But, on the one hand, they always appear in that branch of geometry as *subsidiary steps*, leading to the proof of other theorems which *are* functional, and this seems to be their only scientific relation in that branch; while, on the other hand, the distinction which they suggest between *theorems of dimension* and *theorems of form* has the most essential significance in settling the real difference between the Ancient Geometry and that of Descartes. For, in truth, it may be said that the so-called Pure Geometry is unable to reduce the conception of Form to the functional relation; that its principles and methods are only adequate to the task of befunctioning magnitudes by boldly taking Form for granted; and that the solution of the more general problem, of expressing Form itself in the idea of a function, is only reached in Analytic Geometry, where, by the simple but universally-sweeping "Convention of Coördinates," the Form of every conceivable curve or surface is brought within the conception of a relation between magnitudes, and accordingly represented by an equation. And thus we learn that Geometry in General breaks up into two sub-sciences — *Pure Geometry*, whose functions are those of *Magnitude* merely; and *Analytic Geometry*, whose functions are functions of Form.

Within the province of Pure Geometry we have the subordinate one of *Trigonometry*, in which the theory of the functional character of geometric relations is simply carried out to the last detail in respect to the calculations of the parts of a triangle. But closely allied to this Functional Pure-Geometry, there exists another, antagonistic in idea, whose aim is to carry geometry out of relation to the computative side of mathematics, and set it up as a self-contained science. I refer to what may be called *Constructive Geometry*, with its subdivisions, *Graphics* and *Descriptive Geometry*. The object of these is, to determine by a proportional diagram, whose unit is taken at will, the total linear or superficial value of a required part of a plane figure or a solid, without calling in the aid of any calculation whatever—the required part being taken directly from the drawing (which has been carefully constructed to express the determining conditions), and compared with a standard physical unit. This method has certain ad-

vantages, being sometimes rapid as compared with calculation, and is to a certain extent exceedingly useful, as in carpentry and stone-cutting; but it is inaccurate, from the necessary imperfection of drawing-instruments, and is limited to a comparatively small number of theorems. The method of *Graphics* — which is the constructive solution of *plane* problems—may be described as *direct:* by it, the required part *itself* is drawn and measured; *e. g.*, the line which forms a geometric mean between two given ones; or, the square equal to the sum of any number of given squares. The method of *Descriptive Geometry*, on the contrary, is *indirect:* its object is the construction of the parts of *solid* figures, which cannot be directly drawn on a flat surface: hence the brilliant invention of MONGE, by which we substitute, for the parts of figures themselves, their *projections* (or *shadows cast from an infinite distance*) upon auxiliary planes; these projections once constructed according to a systematic method which Monge has developed, we readily pass from their dimensions to those of the actual figures, with which they are connected by a uniform and very simple functional law.*

The system of Graphics presents itself in three stages:— *Lineal Geometry*, the comparatively recent invention of LAMBERT, in which the constructions are effected with the help of the ruler alone; *Geometric Construction* technically so-called, in which we employ only the ruler and compasses; and *Mechanical Construction*, in which curves of higher orders than the circle are employed, and are generated by means of special instruments which embody some one of their defining properties.

2. The term *Mechanics*, as used to designate the second province of the Mathesis, must be understood as strictly confined to the necessary *a priori* laws of motion and moving systems, and as therefore excluding the vast body of physical considerations and contingent conditions which constitute so large a part of Mechanics as ordinarily understood; indeed,

* The careful reader can hardly fail to notice that the above description of the bearing of Graphics and Descriptive Geometry upon the whole science, is essentially the same as Comte's. Comte, indeed, appears to me to have seized the exact philosophy of this particular episode in his subject, and his treatment of it seems the happiest he has reached.

it might be well to replace it by the term *Kinematics*, were this not so novel and awkward in sound that the mathematical public does not seem to take to it kindly; it is an objection, too, that the new title seems to shut out the consideration of forces in equilibrium. And, in fact, we shall better define Mechanics as the *Science of Force* than as the science of Motion; because, in order to cover systems of motion with entire generality, and so include the system in equilibrium, we must seize Motion as *potential* rather than *actual*, that is, as Force.

The well-known subdivisions of Mechanics—Statics and Dynamics—are thus provided for. In the former, we investigate the laws of Equilibrium, which pass into the functional form by the process of resolving forces into their components; in the latter, the laws of actualized Motion, whether simple or systematic.

II. Crossing now to the CALCULUS, or the computative side of mathematics, it is plain that here, too, there is a great twofold division: assuming that the Mathesis furnishes to our hand the numerical laws which connect required values with given ones, these laws must be expressed in proper symbols —in a word, they must have a convenient *algorithm*— and, when so expressed, the resulting equations (for a numerical law cannot be expressed except in an equation) may require transformation of different orders before they become available for the actual calculation of the values required; finally, when they *are* so transformed, the fundamental processes of number must be applied to them, and the required value thus actually found. Hence the Calculus breaks up into (1) the *Transformation of Functions*, and (2) the *Evaluation of Functions*.

For, defining a function as a *number derived from others by certain numerical combinations, so that its value varies with theirs*, we must distinguish between its *value* and what may aptly be called its *form*. By its *value*, is meant its *place in the general scale of numbers*, or the *result* of the operations by which it is derived from its primitives; by its *form*, the series of *operations themselves* by which it *is* derived. Thus, in the familiar equation $y = \sqrt{2rx - x^2}$, which connects

the length of the perpendicular dropped from the circumference upon the diameter with that of the two segments into into which it divides the latter, the *value* of the function y is the number for which y stands; while its *form* consists in the operations by which that number is derived from the number x, namely, multiplying x by the diameter $2r$, subtracting the square of x from the product, and extracting the square-root of the remainder.

In view of this distinction, then, we may define *Arithmetic* as the *Science of the Evaluation of Functions;* and to the *Science of the Transformation of Functions* we may, in accordance with the uniform usage of such philosophic mathematicians as LAGRANGE and HAMILTON, give the name of *Algebra.**

1. Of *Arithmetic* it needs only to remark, that this branch includes the general Theory of Numbers, and the methods of rapidly combining, in the so-called "fundamental operations," the essential properties developed by that.

2. To render clear the grounds for subdividing *Algebra* into a Lower and a Higher Calculus, we must distinguish between the two states, *implicit* and *explicit*, in which the form of a function may exist, and the two classes, *commensurable* and *incommensurable*, into which the profoundest analysis finally resolves numbers. For, on the one hand, a function as brought to us from the Mathesis may be mixed up in various terms of the equation which connects it with its primitives, and then its form is only *implicit;* or it may stand alone and clear in the first member of the equation, while its primitives in their due combination make up the second, and then its form is *explicit*. And, on the other, just as numbers are primarily conceived as parted from each other by a constant finite interval called *unity*, and are in general simply exact integral or fractional multiples of that, while yet we everywhere find numbers, like 2 and 3, whose *ratio* (or *relative* complex) of this unity we can never find, but can only approximate endlessly; so, in attempting to reduce Extension or Motion to Number, by the aid of an arbitrary unit of their own species, we may

* Lagrange, *Leçons sur le Calcul des Fonctions*, leçon 1; Hamilton, *Algebra as the Science of Pure Time*, Transac. Royal Irish Acad., vol. xvii.

and often do come upon quantities that result only in these *incommensurable* numbers; *e. g.*, the ratio of the circumference of a circle to the diameter. The secret of such numbers seems to be, that, from the very nature of Number in general, there is no limit to the smallness of the "unit," and these incommensurables simply push the mind back again upon that *continuity* for whose cancellation the Understanding so eagerly seized upon the contrivance of the "unit," but which persists through all as the essential principle of quantity itself. Since, then, the comparison of the parts of Extension and Motion must inevitably lead to functions expressible only in terms of incommensurables, it becomes an essential to the perfection of the calculus that we shall be able to transform such relations between incommensurables into corresponding ones between commensurable, or ordinary discrete numbers. Moreover, as in comparing two incommensurable magnitudes, such as the diameter and the circumference, we endlessly approach the true ratio as we go farther and farther on in the process of measuring the greater by the less, the less by the remainder, the remainder by the new remainder, and so on; we see that the logical unit of this tantalizing ratio is an evasive, endlessly diminishing unit—an *infinitely small element* of the magnitudes compared—in a word, a *differential.* Hence, the added necessity that our calculus should enable us to pass readily from finite quantities to these their infinitesimal elements.

In receiving, then, a function to be transformed, it may either be our object to devise a method of passing at will from the implicit to the explicit state of its form, in order directly to evaluate it; or, to discover a general method of passing from finite functions to their infinitely small elements, and reciprocally, in order to sweep the case of treating commensurable relations into the more generic one of treating incommensurable; in short, our object may be either to find the exact *discrete* numerical law which constitutes the *form* of our function, or the *continuous* numerical law which constitutes the *element-form* of the function; and thus the whole calculus does break up into a Lower and a Higher; the former having for its object that order of transformations which ends in making the function *explicit* indeed, but leaves the *law* of

the form unchanged; and the latter, that order which enables us to interchange the *form* and its *element-form*, passing in general from one law of derivation to another entirely different and more manageable.*

We thus define the Lower Calculus (*Ordinary Algebra*) as the *Calculus of Equivalent Forms*, whose object is to interchange the implicit and explicit state of a function; and the Higher Calculus as the *Calculus of Element-Forms*, whose object is to interchange the finite and infinitesimal form of a function. But this transcendental transformation may take place upon two conditions regarding the finite form: either this may be regarded as *constant*, or it may be viewed as *variable* — undergoing a perpetual and continuous change, our problem being to determine what constant form it must assume in order to satisfy certain conditions. And this last distinction points out, in the most generic way, the difference between the Differential and Integral Calculus on the one hand, and the Calculus of Variations on the other. So that we define the Differential and Integral Calculus as the *Higher Calculus of Constant Forms*, and the Calculus of Variations as the *Calculus of Variable Forms*.

The details of this scheme are recapitulated in the table annexed. I hope that it covers the ground with entire generality. That the New Geometry, the Modern Algebra, and the Quaternion, have no distinct place in it, is simply because, as it thus far seems to me, they are but modifications of the same general ideas for which the scheme already provides. Certainly the New Geometry has no logical ground distinct from that of the old—it is merely a group of new and remarkable theorems, some of which have such a sweeping generality as to become the fruitful sources of a *method* of demonstration. Likewise, the Modern Algebra is merely a modification —on a higher plane, to be sure—of the ideas constituting our ordinary algebra: it is a contrivance for the more rapid passage from the implicit to the explicit in complicated cases.

* Thus, the ratio of the semi-circumference to the diameter being incommensurable, we cannot express the one as a finite function of the other. But the *differential* of the semi-circumference *is* a finite function of the *differential* of the diameter, and its form is the very simple one, $ds = dx \left\{ \frac{r^2}{2rx - x^2} \right\}^{\frac{1}{2}}$

And the Quaternion itself, so far as I can at present judge of its logical character, is also within the scope of the wide conception of Algebra as the Science of Transforming Functions: it appears to differ from the present Algebra in what its algorithm connotes—as it represents not merely magnitude (as does the present Algebra), but magnitude, direction, and position at once.

I ought not to close this article without alluding to the points which it still leaves as desiderata. It proceeds upon an assumption that our familiarity with the concept Quantity may be taken for granted; and so, also, of the notions of Space and Time. A perfect dialectic of the subject would have cleared up this obscurity, and shown us the precise logical nature of these thought-elements: this work awaits execution, unless we accept the proffered dialectic of Hegel. I am obliged to acknowledge that I have not yet been able to see that we must do so.

MATHEMATICS, THE FORM OF INTELLIGENCE IN WHICH WE SEEK TO BRING PHENOMENA UNDER THE CATEGORY OF NUMBER,

MAY BE SUBDIVIDED INTO

A. SPECULATIVE = the Science of the Functional Laws by which to convert Figure and Force into Number.

I. THE MATHESIS = the Science which *creates* Functions, by establishing the Laws of Derivation.

1. *Geometry* = the Science of Figure:

A. Computative, in which the Derivation of Quantities is left to be effected by *Calculation*.

a. Pure Geometry = the Befunctioning of *Magnitude*.

b. Analytic Geometry = the Befunctioning of *Form*.

B. Constructive, in which Derivation is effected by *Proportional Drawing*.

a. Graphics = Constructive Geometry of Plane Figures; or, *Direct* Constructive Geometry.

α. Lineal Geometry, which employs the Ruler only.

β. Geometric Construction, which employs the Ruler and Compasses.

γ. Mechanical Construction, which employs the Instruments generating Higher Curves.

b. Descriptive Geometry = Constructive Geometry of Solid Figures; or, *Indirect* Constructive Geometry.

2. *Mechanics* = the Science of Force:

A. Statics = the Science of *Equilibrium*.

B. Dynamics = the Science of *Motion*.

II. The Calculus = the Science which *computes* Functions, either in *Form* or in *Value*.

1. *Algebra* = the Science of the *Transformation* of Functions:
   - A. Lower Calculus (*Common Algebra*) = Calculus of *Equivalent Forms*, whose object is to pass from the *Implicit* to the *Explicit* state of a Form.
   - B. Higher Calculus = Calculus of *Element-Forms*, whose object is to interchange Finite and Infinitesimal Forms.
     - *a*. Differential and Integral Calculus = Higher Calculus of *Constant* Forms.
     - *b*. Calculus of Variations = Higher Calculus of *Variable* Forms.
2. *Arithmetic* = the Science of the *Evaluation* of Functions:
   - A. Theory of Numbers.
   - B. Theory of Operations.

B. APPLIED = the Use of the Science in reducing Phenomena to Number.

I. Astronomy:
1. *Celestial Mechanics.*
2. *Calculus of Observations.*

II. Surveying:
1. *Geodesic.*
2. *Topographical.*
3. *Mensurative.*

III. Navigation:
1. *Celesto-Spherical* (Nautical Astronomy).
2. *Plane* (Sailing by Dead Reckoning).

IV. Physico-Mathematics:
1. *Acoustics.*
2. *Optics.*
3. *Thermodynamics.*
4. *Molecular Mechanics.*
5. *Physico-Mechanics:*
   - A. Theoretical — of Rigids = Statics and Dynamics.
     — of Fluids = Hydrostatics and Hydrodynamics.
   - B. Practical:
     - *a*. Architecture, with the subsidiary art of Stereotomy.
     - *b*. Engineering.
       - α. Civil — Mining, Road-Building, Bridging.
       - β. Military — Fortification, Gunnery.
       - γ. Naval — Ship-modelling, Armoring.
       - δ. Mechanical — Calculations for Machinery and Motors.

THE PHILOSOPHY OF ARISTOTLE.

Translated from the German of G. W. F. HEGEL.

II.—THE METAPHYSICS.

The speculative idea of Aristotle is to be gathered principally from the books of The Metaphysics, the main source being the last chapter of the 12th Book (*Λ*), which treats of divine thinking. But this work has, as we have before mentioned, a quite peculiar difficulty in its composition: several writings have been pieced together, and this work is the result. Aristotle and the ancients do not know this work at all under the present name of Metaphysics, but they call it *Πρώτη Φιλοσοφία* [*Prima Philosophia*]. Or if the body of this work is one, as seems from the general connection,* at all events it cannot be said to be well arranged and clearly written. This pure philosophy is defined by Aristotle (Met. IV. 1 [or Book III. Bohn's tr.]) and carefully discriminated from the other sciences as "The Science of what exists, in so far as it exists [i. e. in its totality], and what pertains to it in-and-for-itself." To know scientifically the nature of this substance (*οὐσία*) is Aristotle's chief endeavor (Met. VII. 1, [Bohn VI.]) In this Ontology, or, as we call it, Logic, he investigates and defines accurately the four principles (Met. I. 3): first, the determinateness or QUALITY, as such, through which something is this, the ESSENCE or FORM; secondly, the MATTER; thirdly, the principle of MOTION; and fourthly, the principle of FINAL CAUSE or of THE GOOD. In the course of this work Aristotle refers frequently to the definition of these ideas; but in this work, as in others, all is arranged in a loose manner, although it is united to a genuine speculative insight.

Next we have two chief forms, which Aristotle defines as POTENTIALITY (*δύναμις*) and ACTUALITY (*ἐνέργεια*); afterwards still more definitely as ENTELECHY (*ἐντελέχεια*), or free activity which has its FINAL CAUSE (*τὸ τέλος*) in itself and is the realization of this final cause. These are determinations [defini-

* Professor C. L. Michelet, the editor of this work of Hegel, has discussed these questions fully in his work, "*Examen critique de l'ouvrage d'Aristote, intitulé Metaphysique.*"

tions or distinctions] that occur everywhere in Aristotle, and are discussed *in extenso* in the 9th Book of the Metaphysics [Book VIII. Bohn's tr.], and one must learn them in order to comprehend Aristotelian philosophy. The expression δύναμις in Aristotle is the natural capacity, the In-itself [potentiality], the objective; secondly, it is the abstract Universal as such, the idea; the matter which can assume all forms without being itself the formative principle. With such an empty abstraction as the "Thing in itself" Aristotle has nothing to do. The Energy [Actuality], or more concretely the Subjectivity, is the actualizing form, the negativity which relates to itself. When we, on the other hand, use the term "Essence," we do not posit [express] activity in it; but Essence is only the In-itself, only potentiality without infinite form ["infinite" means *self-related*]. The chief point in the definition of "substance" is, according to Aristotle, that it is to be distinguished from mere *matter* (Met. VII. 3); in common life matter passes ordinarily for what is substantial. Whatever exists, contains matter; all change implies a substrate (ὑποκείμενον); for the reason, however, that matter itself is only potentiality, and not the Actuality which belongs to the Form, the Activity of the form is essential to the matter in order that it [i. e. matter] may possess reality (Met. VIII. [VII.] 1–2). With Aristotle, therefore, δύναμις does not have the signification of Force at all (force is rather an imperfect phase of *Form*), but rather *capacity*—but not in the sense of undetermined possibility; ἐνέργεια, however, is the pure self-activity [spontaneity]. Through the entire middle ages these distinctions were considered very important. According to Aristotle, therefore, the essential absolute substance has potentiality and actuality, form and matter; these are not separate from each other, but the true Objective possesses also activity in itself, while the true Subjective has also potentiality.

From this exposition becomes clear the nature of the antithesis between the Platonic and Aristotelian Ideas. For, although the Idea in Plato is in itself essentially concrete and determined [definite], yet Aristotle goes further [in defining it]. In so far, namely, as the Idea is self-determined, the relation of its moments can be more definitely stated; and

this relation of moments to each other is to be apprehended as nothing else than Activity. With us it is easy to make understood what is meant by the defect [lack or onesidedness] of the Universal, i. e. the defect of that which is only *in-itself* [or potential]. The Universal has, since it is universal [merely general], as yet ["so far forth"] no actuality; for since the In-itself [potentiality] is inert, the activity of the process of actualization is not yet posited [or expressed] in it. Reason, laws, &c., are in this sense abstract; but the Rational as self-actualizing we recognize as necessary [or overruling power]: and therefore we do not lay much stress on such general laws. The Platonic standpoint expresses the essence more as the objective, the good, the final cause, the Universal in general; in this, however, there is lacking the principle of living [vital] Subjectivity as the moment of actuality; or, at least, this principle seems to remain fixed in the background, and in fact this negative principle is not expressed by Plato in so immediate a form, although it is contained essentially in his doctrine that defines the Absolute as the unity of opposites; for this unity is essentially *negative unity* of these opposites which cancels [annuls] their other-being, their opposition, and reduces them to self-relation. But Aristotle defines in a precise manner this negativity, this active process of actualization, as *Energy:* its nature is to cancel this unity and to dirempt [i. e. duplicate itself] this being-for-itself, and to posit the diremption; for, as Aristotle says (Met. VII. 13), "The Entelechy sunders." The Platonic idea, on the contrary, is rather that cancelling of the opposites wherein one of the opposites is itself the unity. While therefore, with Plato, the affirmative principle, the idea seized as the mere abstract self-identity, is taken for the highest; on the other hand, with Aristotle, the moment of negativity, not as change, nor as nothing, but as distinction [or difference], determination, is reached and brought into use by him. This principle of individualization, not in the sense of a contingent and merely special subjectivity, but of the pure subjectivity, is the peculiar doctrine of Aristotle. Aristotle, too, holds the Good taken as the universal end, to be the substantial principle, and holds it in opposition to Heraclitus and to the Eleatics. "The Becoming" of Heracli-

tus is a correct, essential determination; but the category of Change lacks the determination of identity with itself, the solidity of universality. The stream is perpetually changing; and yet it is perennial, and is still more than this, a universal existence. From this, then, it is clear that Aristotle contends (Met. IV. 3–6) particularly against Heraclitus and others when he says that being and non-being are not the same; and with this founds the well-known principle of contradiction [which is illustrated by the assertion] that a man is not a ship. It is evident that Aristotle does not intend to apply this remark to *pure* being or non-being, this abstraction which is essentially only the transition of the one into the other; but under that *which is*, he understands essentially *substance*, *idea*, *Reason*, but in the sense of an active final cause; while, on the one hand, he sets up the Universal against the principle of mere change, so too, on the other hand, he defends the principle of *activity* against the Pythagorean system of numbers and the Platonic system of ideas. Although Aristotle polemicizes against both in many places, yet the objections which he urges, all hinge on one point [already quoted by Hegel in speaking of Pythagoras]: The activity is not to be found in these principles; that "actual things participate in ideas" is an empty expression or a poetical metaphor. The ideas, as abstract universal determinatenesses, are according to number just as numerous as the things, but cannot for this reason be exhibited as their causes. Further contradictions are involved by assuming independent genera, because in Socrates, e.g., there are several ideas combined: man, biped, animal (Met. I. 7 & 9). Aristotle's category of "activity" is change, but a change posited within the Universal, change remaining self-identical; consequently a determining which is self-determining, and therefore the self-realizing, universal, FINAL CAUSE; in mere Change, on the contrary, [the conception of] self-preservation is not necessarily involved. This is the chief doctrine added to Philosophy by Aristotle.

Aristotle distinguishes manifold moments in substance, inasmuch as the moments of activity and potentiality manifest themselves not as united in one, but as separated. The full determination of this relation of energy to potentiality,

of form to matter, and the movement of this antithesis gives the different *forms of substance.* Here Aristotle goes through the list of substances; and they appear as a series of different kinds of substances which he takes up in succession rather than brings together into a system. The chief determinations are the following three moments:

a. The substance sensuously perceived is that which has a matter differing from its actual form. This substance is consequently finite; for the separation and externality of form and matter to each other constitutes precisely the nature of the Finite in general. The sensuous substance, says Aristotle (Met. XII. 2), contains change, but in the form of transition into its opposite. The opposites vanish in each other; the third, which preserves itself from this opposition, the permanent under this change, is the Matter. The chief categories of change which Aristotle names are the four distinctions; according to the What (*κατὰ τὸ τί*), Quality (*ποιόν*), Quantity (*ποσόν*), or the Where (*ποῦ*). The first change is the beginning or ceasing of the simple, definite essence (*κατὰ τόδε*). The second change is that of the properties (*κατὰ τὸ πάθος*); the third, increase and diminution; the fourth, movement. Matter is the dead substrate on which the changes that it suffers take place. "Change is from the *potential* to the *actual;* the potential white is changed into the actual white. So that things do not arise from nothing, by chance; but everything begins from something which is not actual but only potential." The "potential" means, therefore, the general In-itself-existent which produces these determinations without developing the one from the other. Matter is the simple potentiality, which however is self-opposed; so that something only becomes in actuality what its matter was in potentiality. There are, therefore, three moments posited: (1) Matter as the universal substrate of change, indifferent to the antithesis involved in change (*ἐξ οὗ*); (2) the antithetic determinatenesses of form, which are negatives towards each other as annulling and positing (*τι* and *εἶς τι*); (3) the first Mover (*ὄφ' οὗ*) the pure activity (Met. VII. 7; IX. 8; XII. 3). [*Remark by Professor Michelet, the editor:* "Not only the form in its annulling activity but also the matter is called *τι* by Aristotle, since in fact the annulled form serves as mere

material for the positing; so that he calls in the first passage the three moments ἔκ τινος, τι, ὑπό τινος, in the last place τι, εἷς τι, ὑπό τινος."] But the activity is the unity of form and matter; how the last two are united in the first, Aristotle does not explain. In this manner appears on the sensuous substance the difference of the moments, but not as yet their return into themselves; but the activity is the negative which contains the antithetic sides in itself ideally, and hence it is already the goal of the process.

b. A higher kind of substance, according to Aristotle (Met. IX. 2; VII. 7; XII. 3), is that which already contains the activity which is the object of its process. This is the in-and-for-itself determined Understanding whose content is the final cause which it actualizes through its activity without undergoing change like the mere sensuous substance. For the soul is essentially entelechy, a general process of determination which posits itself; not a merely formal activity whose content comes from elsewhere. But since the active process posits its content in the actuality, the latter remains the same; it is, however, still an activity different from the matter, although the substance and the activity are united. Thus we have here another sort of matter which the Understanding demands as a presupposition. The two extremes are *matter*, as potentiality, and *thought*, as actuality; the former the passive Universal, the latter the active Universal; in the sensuous substance, on the contrary, the Active is conceived as quite different from the matter. In the former two moments change does not take place; for they are the in-itself-existent Universal in an antithetic form.

c. The highest point is, however, rather that in which *potentiality*, *activity*, and *entelechy* are united,—the ABSOLUTE SUBSTANCE which Aristotle (Met. XII. 6–7; IX. 8) defines in general thus: The in-and-for-itself existent (ἀίδιον) is unmoved [by others], but at the same time it moves [through itself], and its essence is pure activity without having matter. For matter as such is the passive upon which change takes place, and it therefore is not as yet one with the pure activity of this substance. Here we see indeed, as before, how a predicate can be denied without saying what its truth is; matter is, however, only that moment of the unmoved essence [i. e.

it is the unmovedness of the essence taken abstractly]. If in modern times it has seemed a new thing to define the Absolute Essence as pure activity, we see that this happened only through ignorance of the Aristotelian philosophy. The scholastics have rightly viewed this as the definition of God, and have applied to Him the designation of *actus purus;* there is no higher idealism than this. We might express this also in the following manner: God is the substance that contains within His potentiality also His actuality inseparably united; in Him the potentiality is not distinct from the form, since He produces His content-determinations from Himself. In this Aristotle differs from Plato, and from this position he polemicizes against the principle of number, and against the idea and the Universal; because if the latter is inert, it is not identical with actuality, is no movement. The inert ideas and numbers of Plato thus bring nothing into actuality; but the Absolute of Aristotle is in its repose at the same time an absolute activity.

More in detail, Aristotle remarks (Met. XII. [XI.] 6) upon this subject: "It can happen that what has potentiality is not actual; for this reason it does not help us any to make substances eternal, as does the doctrine of ideas, if they do not contain a principle which admits of change. And, moreover, it would not avail anything if it did possess such a principle unless it were active; for then there would be no change. Nay, even were it active, if its substance were only a potentiality, then there could be no eternal motion; for potentiality admits of existence and non-existence. Hence there must be a principle whose substance is to be apprehended as activity." Thus in Spirit the substance is energy itself. "There seems at this point, however, to arise a doubt. For all activity seems to be possible, but not all possible to energize [i. e. not every potentiality is actual]; so that the potential seems to be primary [or antecedent to energy]," for it is the Universal [general]. "But if it were so, no being would exist; for it is possible that a thing *might* be although it is not. But energy is higher than potentiality. It cannot, therefore, be said, as the Theologians affirm, that there was first in infinite past time a chaos or night" (matter), "or, as the Physicists hold, that all existed simultaneously [i. e. all is eternal].

How could this first [chaos or matter] be changed if nothing actual were the cause? for matter cannot move itself, but only the workman can move it. Therefore Leucippus and Plato assert movement to be eternal, but they do not say why." The pure activity is, according to Aristotle (Met. IX. 8), prior to potentiality, not according to time, but according to essence [logically, not chronologically, prior]. Time is a subordinate and is not a constituent element of what is universal; for the absolutely primal Essence is, as Aristotle says at the close of the 6th chapter of the 12th Book, "that which abides self-identical in equal [self-consistent] actuality." In that assumption of a chaos, &c., an actuality is posited which becomes another, not itself. and thus has a presupposition [i. e. it presupposes the other as its possibility]; chaos is, however, only the empty potentiality.

The true essence is to be posited, continues Aristotle, as that "which moves in a circle" (Met. XII. 7); "and this is not alone to be seen in the thinking reason, but also in the fact." From the determination of the Absolute essence as active which makes it issue forth into actuality, it follows that it exists in an objective manner in visible nature. As the self-identical which is visible, this absolute essence is "the *eternal heavens*"; the two modes of exhibition of the Absolute are thus the thinking Reason and the eternal heavens. But the heavens are [passively] in motion: they should, however, be also an active mover. While the spherical form is thus moving and moved, there is a middle term which moves but is the unmoved, and is at the same time eternal and a substance and an energy.* This great doctrine of Aris-

* *Note by Prof. Michelet.*—"Since this Hegelian exposition of the famous Aristotelian passage has so many authorities for it, the editor did not venture to follow the practice of his associate editors here, as he has done elsewhere frequently in these lectures, and change inaccuracies silently. It is clear that Aristotle speaks in this place of three substances: (1) A sublunary world which is moved by the heavens; (2) the heavens as the middle, which are at once mover and moved; (3) and God the unmoved mover. The passage must be punctuated according to Alexander Aphrodisias (Schol. in Arist. Ed. Brandis, p. 804, *b*), Cardinal Bessarion (Aristotle lat. ed. Bekk. p. 525, *b*), and others, so that the translation will read as follows: Besides the perpetually moved heaven, there "is also something which it moves. But since that which is moving and moved at the same time is also a middle term, it follows that there is also an unmoved mover." Cf. Michelet: Exam. Crit. p. 192.

totle, as the circle of the Reason returning into itself, sounds like doctrines that have been reached in modern times; the "unmoved which moves" is the Idea which is in eternal self-identity, which remains in relation to itself while it moves. He explains this in the following way: "Its movement is determined in this manner: that moves which is desired and thought; it is, however, itself unmoved, and the original of both is the same." That is the final cause whose content is desire and thought; such a final cause is the Beautiful or the Good. "For what is desired is that which appears as beautiful" (i. e. which pleases), "whose first" (or final cause) "which is willed by the will is what is beautiful. We desire it, however, because it so appears, rather than that it so appears because we desire it." For then it would be posited only through the [subjective] activity; it is, however, posited as an independent objective essence through which our desire is first aroused. The true principle is in this, however, the thinking [activity]; for thought is only moved by thought. The other co-element, *συστοιχία*,* is however intelligible" (one scarcely believes his eyes) "in-and-for-itself," namely, the in-and-for-itself-existing thought which is posited as objective; "and this substance of the other element is the first; the first substance is, however, the simple, the pure activity. The Beautiful and the Best are precisely similar; and the first [cause] is eternally the absolute, or the best possible. But that the final cause [or the purpose] belongs to the unmoved is evident. What is moved can also be otherwise than it is. The movement (*φορά*) is the first change; the first movement is again the circular movement; this latter is, however, moved by the former." According to Aristotle, therefore, the Idea, the *principium cognoscendi*, is also the mover, the *principium essendi;* he expresses this as God, and shows its relation to the individual consciousness. "The first cause is necessary. *Necessary* is to be predicated in three ways: firstly, the violent [external force] which is against the inclination (*παρὰ τὴν ὁρμήν*); the second, without which the good is not; thirdly, what cannot be in another manner, but is absolutely as it is. To such a principle of the unmoved the heavens are

* *Note by Hegel.*— *Συστοιχία* is a good word; it may signify an element which is its own element, and determines itself only through itself.

attached and the whole of nature,"—the visible eternal and the visible changeable. This system endures always. "For us, however," as individuals, "it is an abode permitted only for a short time, which is most excellent. To the former [i. e. God] it is eternally thus; for us this is impossible. Since, now, His activity is enjoyment itself, it happens on this account that waking, feeling and thinking are the richest in pleasure: hopes and memories are enjoyment [mediately] for the same reason. The thinking, however, which is pure-for-itself is a thinking of that which is the most excellent in-and-for-itself"; the thought is for itself its own absolute final cause. The distinction and antithesis in the activity, and the cancelling of the same, is expressed by Aristotle thus: "The thought thinks itself through participation (*μετάληψιν*) in thought; it is, however, thought through contact and thinking; so that the thought and that which is thought are the same." Thought, since it is the unmoved which moves [causes motion], has an object, which however passes into activity since its content is also what is produced through thought, and hence identical with the activity of thinking. [The object of thought is first begotten in the activity of thinking, which is therefore a separation of the thought as an object. Here, in the thinking, is therefore that which is moved and that which moves the same: since the substance of that which is thought is the thinking activity, that which is thought is the absolute cause which, itself unmoved, is identical with the thought which is moved by it; the separation and the relation are one and the same. The chief moment of the Aristotelian philosophy is therefore this: that the energy of thinking and the object which is thought are one and the same; "for that which apprehends what is thought and the essence, is thought. Its possession is one with its activity (*ἐνεργεῖ δὲ ἔχων*) [or, it is a continuous energy]; so that this" total of activity through which it thinks itself "is more divine than that which the thinking reason supposes to possess that attribute"—i. e. the content of thought. Not that which is thought is the more excellent, but the energy of thinking itself; the activity of the apprehending produces that which is perceived [the total activity is more divine than one phase or moment of it seized abstractly]. "Speculation (*ἡ θεωρία*) is

thus the most delightful and best. If God, now, is always in this, as we are at times" (in whom this eternal thinking, which is God himself, occurs only as individual condition), "then He is admirable; if still more, then more admirable. But He is thus. Life, too, is His; for the actuality of thought is life. He, however, is activity; the activity returning into itself is His most excellent and eternal life. We say, therefore, that God is an eternal and the best life." From this substance Aristotle excludes, moreover, quantity.

We in our language describe the absolute, the true, as the unity of the subjective and objective, which on this account is neither the one nor the other, and as well the one as the other: but Aristotle has likewise expressed himself in these forms which are still to-day the deepest speculative forms, and has defined the Absolute and True in its highest clearness. Hence it is not a dry identity of the abstract understanding that he means, but he distinguishes subjective and objective firmly and strictly. Such dead identity is not the most worthy of honor — it is not the God of Aristotle; but Aristotle finds energy to be this. *Unity* is also a poor, unphilosophical expression, and the true philosophy is not the "system of identity": but its principle is a unity which is activity, movement, repulsion, and hence is identical with itself in its difference. If Aristotle had made the insipid identity of the understanding, or of experience, his principle, he would never have come to such speculative ideas, and would not have posited individuality and activity higher than general potentiality. The thought as that which is thought [i. e. the object] is nothing else than the absolute idea considered *in-itself*—"the Father" [stated theologically]; yet this first, unmoved, as distinct from the activity, is still as absolute the activity itself, and is first posited in its truth when posited through this. In the doctrine of the soul, Aristotle returns again to this speculative thinking; it is in that place, however, to Aristotle a condition to be taken up separately and empirically just like other conditions, such as sleep, weariness, &c. He does not say in that place that it is the only truth, that all is thought; but he says: it is the first, strongest, most honored. We, however, say: that thought as the absolutely self-related *is* the truth; that thought is *all* truth,

though we encounter in ordinary consciousness such activities as *feeling*, &c., as actual, side by side with the thinking. Although Aristotle does not express himself as Philosophy does at the present day, yet there lies at bottom the same insight; he does not speak of a special nature of Reason, but of universal Reason. It is precisely the character of Aristotle's speculative philosophy, to consider all objects in [speculative] thought [i. e. as total processes and not as abstract phases], and thus to elevate them into thoughts [i. e. taking any mere empirical object to bring it before the mind as a whole, and thus to seize it in its universality and necessity]; so that while they are as thoughts [total processes] they are in their truth. By this statement it is not meant, however, that the objects of nature are therefore thinking [or conscious] beings]; but, when they are thought subjectively by me, my thought is also the ideal totality [necessary form of the reality] of the thing, which thus constitutes their in-and-for-itself-existing substance. In nature the Idea exists, however, not for itself as thought in this freedom, but has flesh and blood, and is encumbered with externality; this flesh and blood has, however, a soul, and this is its idea. In the common definition of truth according to which it is "the agreement of the conception with its object," truth does not yet find its complete realization in the "conception"; for if I form a conception of a house or of a timber, I am not this content at all, but something quite different, and my conception is not at all in agreement with the object. Only in thinking is found the true agreement of the objective and subjective; I am identical with thinking. Aristotle is found, therefore, upon this highest standpoint; one cannot desire to know anything deeper, although he always uses the form of beginning from mere conceptions.

Here Aristotle solves (Met. XII. 9) many doubts; such as, e.g., whether thought is compound; or, whether science is the thing itself. "There arise still some doubts concerning thought (νοῦς), which seems to be the most divine among all existences; it has some difficulties in defining the conditions under which (πῶς δ' ἔχων) it is this [i. e. the divinest]. If a man thinks nothing, but conducts himself as one who sleeps, what advantage has he over the latter? If, however, he thinks, but another controls him therein (ἄλλο κύριον), then that which is

his substance would not be the thinking (*νόησις*), but the thinking would be only a potentiality." He would not be in eternal activity. "Then he would not be the best substance; for on account of the (active) thinking (*τοῦ νοεῖν*) he has his worth. Whether now, further, the thought or the thinking activity is his substance, what does he think, himself or another? and if another, always the same, or a different? Does it not make a difference whether one thinks the Beautiful, or only the Accidental? First, now, if the thought is not the thinking [i. e. if mind is not an activity] but is only a faculty [or capacity], the act of continuous thinking would be a tiresome operation to it," for each force wastes itself. "In that case another would be more excellent than thought, namely, the object of thought (*νοούμενον*); and the thinking and thought (*τὸ νοεῖν καὶ ἡ νόησις*) would find themselves in that condition which thinks the most inferior. Since this is to be avoided (as it is better not to see some things than to see them), then thinking would not be the best. Thought is therefore this: to think itself, because it is the most excellent; and it is the thinking which is *thinking of thinking*. For understanding and feeling and opinion and deliberation *seem* always to concern another and to relate to themselves only incidentally. Moreover, if the thinking and being thought are different, in which of the two does the Good appertain to thought? For the ideas of thinking and of that which is thought are not the same. Or is Science in some cases the thing itself [i. e. its own subject-matter]? In the practical, the subject-matter is immaterial substance and the determinateness of the final cause (*ἡ οὐσία καὶ το τί ἦν εἶναι*); in the theoretical it is the ground and the thinking. Since, therefore, that which is thought and the thought thereof are not different, these opposites, in so far as they have no matter, are the same; and there is only one thought of that which is thought." Reason which thinks itself is the absolute final cause, or the Good; for it is only on its own account. "Yet there is still a doubt whether the thinking is not a compound; for in that case it might change in the parts of the whole. The Good is not in this or that part, but it is the best in the universe as another than it [i. e. the Good is found in the totality]. Thus the thinking activity relates eternally to itself.